Assigned: Thursday 4/19 DUE ON WEDNESDAY 4/25, MUST SHOW WORK FOR CREDIT

UNIT 4: MODELING AND ANALYZING EXPONENTIAL FUNCTIONS Name KEY

Question	Answer
Create Equations That Describe Numbers or Relationships	
1. An amount of $1,000 is deposited into a bank account that pays 4% annual interest. If there are no other withdrawals or deposits, what will be the balance of the account after 3 years?	$y=1000(1+.04)^3$ $= \$1124.86$
2. The city of Arachna has a spider population that has been doubling every year. If there are about 100,000 spiders this year, how many will there be 4 years from now?	$y=100000(2)^4$ $= 1,600,000$ spiders
3. A certain population of bacteria has an average growth rate of 0.02 bacteria per hour. The formula for the growth of the bacteria's population is $A_0 = P(2.71828)^{0.02t}$, where P_0 is the original population, and t is the time in hours. If you begin with 200 bacteria, about how many bacteria will there be after 100 hours?	**A.** 7 **B.** 272 [✓ **C.** 1,478] ~~**D.** 20,000~~ $A=200(2.71828)^{.02\cdot 100}$ $=1477.8 \approx 1418$
4. Pete withdraws half his savings every week. If he started with $400, what rule be written for how much Pete has left each week?	$y=400\left(\frac{1}{2}\right)^x$
5. Consider the number of sit-ups Clara does each week as listed in the sequence 3, 6, 12, 24, 48, 96, 192.	**a) Is this an arithmetic or geometric sequence?** geometric **b) What is the explicit formula?** $a_n = 3\cdot(2)^{n-1}$ **c) What is the recursive rule?** $a_n = 2\cdot a_{n-1}$, $a_1 = 3$
6. A scientist collects data on a colony of microbes. She notes these numbers: Day / a_n 1 / 800 2 / 400 3 / 200 4 / 100 5 / 50	**What function can she use to model the population size?** exponential $a_n = 800\cdot\left(\frac{1}{2}\right)^{n-1}$
6. The temperature of a large tub of water that is currently at 100° decreases by about 10% each hour. **Part A:** Write an explicit function in the form $f(n) = a \cdot b^n$ to represent the temperature, $f(n)$, of the tub of water in n hours. **Part B:** A recursive function in the form $f(n) = r(f(n-1))$, where $f(1) =$ 100, can be written for the temperature problem. What recursive function represents the temperature, $f(n)$, of the tub in hour n?	Part A: $f(n) = 100\left(\frac{1}{10}\right)^n$ Part B: $f(n) = \frac{1}{10}\cdot f(n-1)$ $f(1) = 100$

7.

Which function represents this sequence?

n	1	2	3	4	5	...
a_n	6	18	54	162	486	...

×3 ×3 ×3

A. $f(n) = 3^{n-1}$
B. $f(n) = 6^{n-1}$
C. $f(n) = 3(6^{n-1})$
D. $f(n) = 6(3^{n-1})$

8. The points (0, 1), (1, 5), (2, 25), (3, 125) are on the graph of a function. Which equation represents that function?

A. $f(x) = 2^x$
B. $f(x) = 3^x$
C. $f(x) = 4^x$
D. $f(x) = 5^x$

x	y
0	1
1	5
2	25
3	125

×5 ×5 ×5

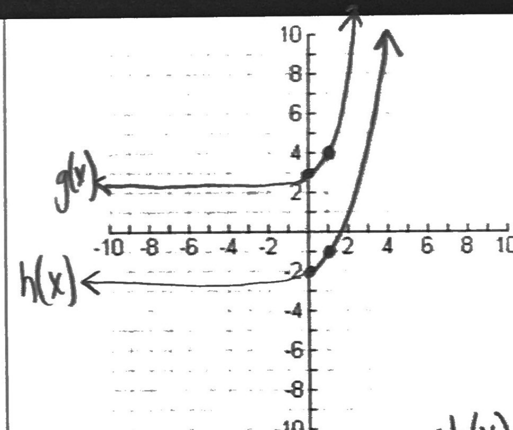
$y = 1 \cdot (5)^x$

Build New Functions from Existing Functions

9. If $f(x) = 2^x$, how will $g(x) = f(x) + 2$ and $h(x) = f(x) - 3$ compare?

$g(x) \to$ shift 2 units up

$h(x) \to$ shift 3 units down

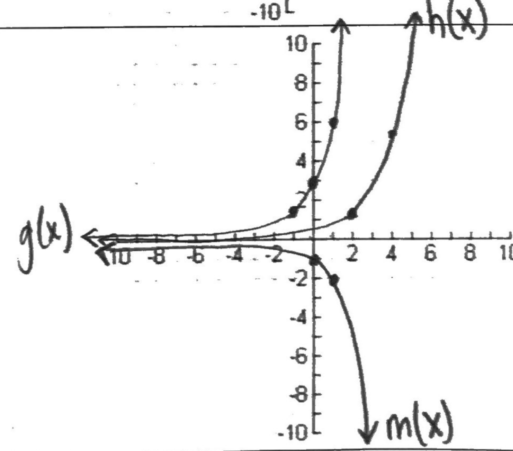


10. If $f(x) = 2^x$, how will $g(x) = 3f(x)$, $h(x) = \frac{1}{3}f(x)$, and $m(x) = -f(x)$ compare?

$g(x) \to$ vertical stretch by 3

$h(x) \to$ vertical shrink by $\frac{1}{3}$

$m(x) \to$ reflection over x-axis



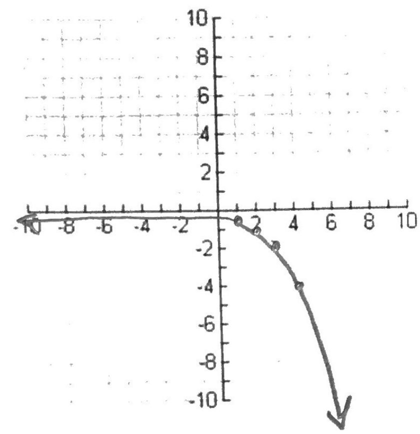
11. For the function $f(x) = 3^x$, find the function that represents a 5 unit translation up of the function.

$f(x) = 3^x + 5$

12. Given the function $f(x) = 2^{(x-2)}$, complete each of the following:

a. Compare $f(x)$ to $3f(x)$. Vertical stretch by 3

b. Draw the graph of $-f(x)$.



c. Which has the fastest growth rate: $f(x)$, $3f(x)$, or $-f(x)$?

13. Which function shows the function $f(x) = 3^x$ being translated 5 units to the left?

A. $f(x) = 3^x - 5$
B. $f(x) = 3^{(x+5)}$
C. $f(x) = 3^{(x-5)}$
D. $f(x) = 3^x + 5$

14. Which function shows the function $f(x) = 3^x$ being translated 5 units down?

A. $f(x) = 3^x - 5$
B. $f(x) = 3^{(x+5)}$
C. $f(x) = 3^{(x-5)}$
D. $f(x) = 3^x + 5$

Understand the Concept of a Function and Use Function Notation

15. Given $f(x) = 2(3)^x$, find $f(7)$.

$f(7) = 2(3)^7 = 4374$

16. If $g(6) = 2^{(6)} + 1$, what is $g(x)$?

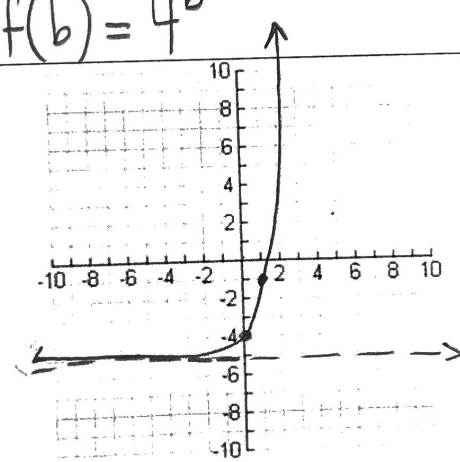
$g(x) = 2^x + 1$

17. If $f(-2) = 4^{(-2)}$, what is $f(b)$?

$f(b) = 4^b$

18. Graph $f(x) = 4^{(x)} - 5$.

x	y
-1	-4.75
0	-4
1	-1
2	11



19. A population of bacteria begins with 2 bacteria on the first day and triples every day. The number of bacteria after x days can be represented by the function $P(x) = 2(3)^x$.

a. What is the common ratio of the function?
$r = 3$

b. What is a_1 of the function? $a_1 = 2$

c. Write a recursive formula for the bacteria growth. $a_n = 3 \cdot a_{n-1},\ a_1 = 2$

d. What is the bacteria population after 10 days?

$P(10) = 2(3)^{10} = 118{,}098$ bacteria

20. **Consider the first six terms of the following sequence: 1, 3, 9, 27, 81, 243, . . .**

a. What is a_1? What is a_3? $a_1 = 1$, $a_3 = 9$

b. What is the reasonable domain of the function?

(+) real integers

c. If the sequence defines a function, what is the range?

$\{1, 3, 9, 27, 81, 243 \ldots\}$

d. What is the common ratio of the function?

$r = 3$

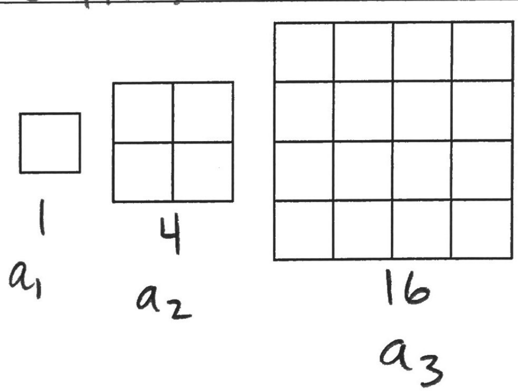
21. **The function $f(n) = -(1 - 4^n)$ represents a sequence. Create a table showing the first five terms in the sequence. Identify the domain and range of the function.**

n	f(n)
1	3
2	15
3	63
4	255
5	1023

Domain: (+) real integers

Range: $\{3, 15, 63, 255, 1023 \ldots\}$

22. **Consider this pattern. Which function represents the sequence that represents the pattern?**



A. $a_n = (4)^{(n-1)}$

B. $a_n = (4)^{(a_n - 1)}$

C. $a_n = (a_n)(4)^{(n-1)}$

D. $a_n = (a_n)^4$

23. **Which function is modeled in this table?**

A. 1,000(0.80)

B. 1,000(0.20)

C. $1{,}000(0.80)^x$

D. $1{,}000(0.20)^x$

x	f(x)
1	1000
2	800
3	640
4	512

24. **Which explicit formula describes the pattern in this table?**

A. $C = 6d$

B. $C = d + 6$

C. $C = 6^d$

D. $C = d^6$

d	C
0	1
1	6
2	36
3	216

25.

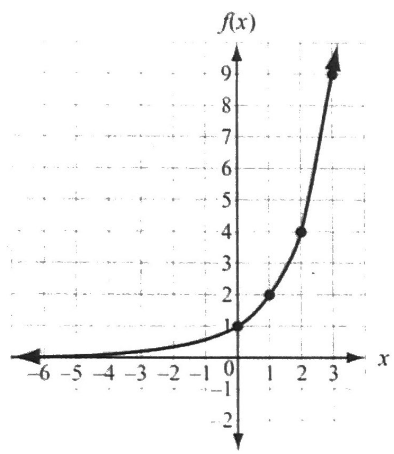
If $f(12) = 100(0.50)^{12}$, which expression gives $f(x)$?

A. $f(x) = 12^x$
B. $f(x) = 100^x$
C. $f(x) = 100(x)^{12}$
D. $f(x) = 100(0.50)^x$

Interpret Functions That Arise in Applications in Terms of the Context

26. Find the following features of $f(x) = 2^x$.

Exponential Function
$f(x) = 2^x$



Domain: $\mathbb{R}$

Range: $y > 0$

x-intercept: none

y-intercept: $(0, 1)$

Interval of Increase: $\mathbb{R}$

Interval of Decrease: none

Maximum: none

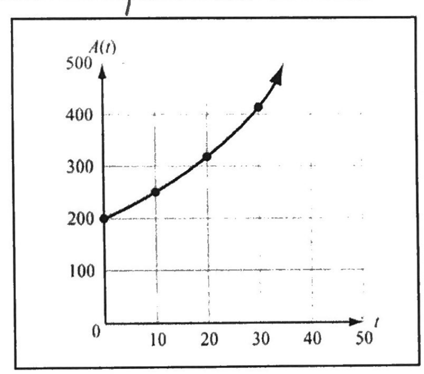
Minimum: none

Rate of Change over [0,1]: ROC

$f(0)=1$
$f(1)=2$
$\frac{2-1}{1-0} = \frac{1}{1} = \boxed{1}$

Asymptote: $y=0$

27. The amount accumulated in a bank account over a time period t and based on an initial deposit of \$200 is found using the formula $A(t) = 200(1.025)^t, t \geq 0$. Time, t, is represented on the horizontal axis. The accumulated amount, $A(t)$, is represented on the vertical axis.

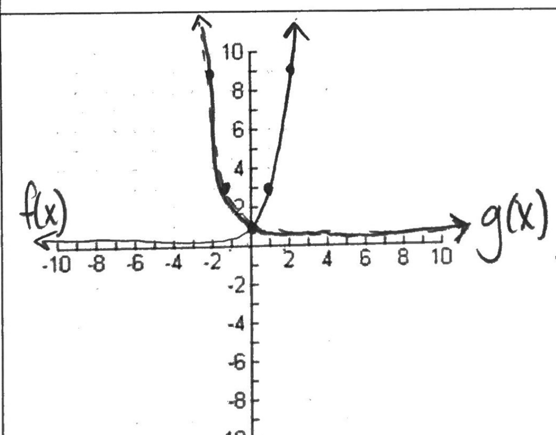


a. What are the intercepts of the function? $(0, 200)$
b. What is the domain of the function? $t \geq 0$
c. Why are all the t values non-negative? you can't have negative time
d. What is the range of the function?

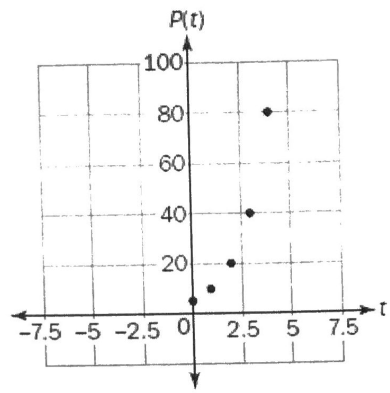
$y \geq 200$

28. Consider two exponential functions, If $f(x) = 3^x$ and $\cancel{f(x)}$ $g(x)$ $= \left(\frac{1}{3}\right)^x$. Compare the key features of the two functions.

f(x)	g(x)
Domain: $\mathbb{R}$	Domain: $\mathbb{R}$
Range: $y > 0$	Range: $y > 0$
x-intercept: none	x-intercept: none
y-intercept: $(0, 1)$	y-intercept: $(0, 1)$
Interval of Increase: $\mathbb{R}$	Interval of Increase: —
Interval of Decrease: —	Interval of Decrease: $\mathbb{R}$
Asymptote: $y=0$	Asymptote: $y=0$

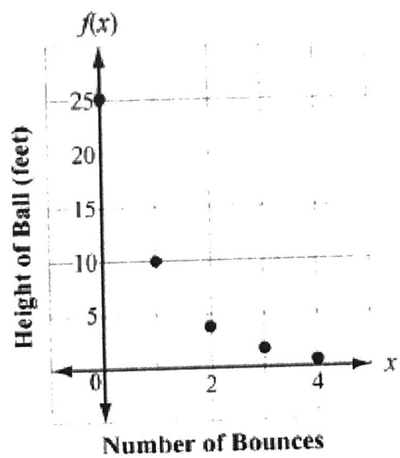


29. **A population of squirrels doubles every year. Initially, there were 5 squirrels. A biologist studying the squirrels created a function to model their population growth: P(t) = $5^{(2t)}$, where t is the time in years. The graph of the function is shown.**



What is the range of the function?
A. any real number
B. any whole number greater than 0
C. any whole number greater than 5
D. any whole number greater than or equal to 5

30. **The function graphed on this coordinate grid shows f(x), the height of a dropped ball in feet after its xth bounce. On which bounce was the height of the ball 10 feet?**

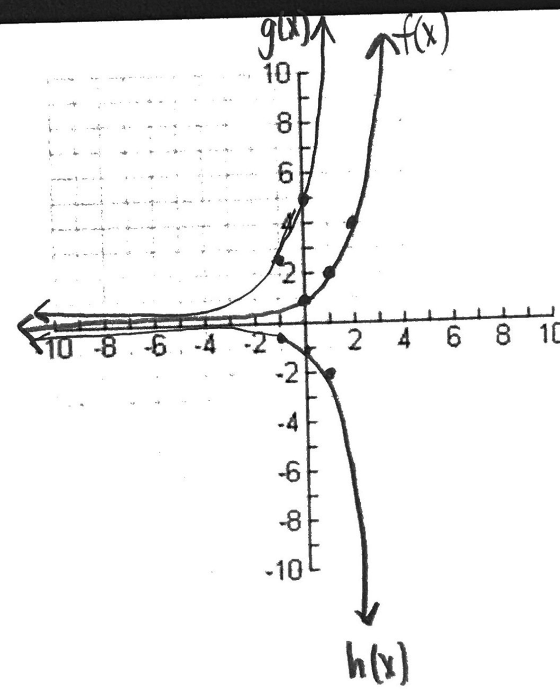


A. bounce 1
B. bounce 2
C. bounce 3
D. bounce 4

Analyze Functions Using Different Representations

31. Consider $f(x) = 2^x, g(x) = 5 \cdot 2^x$, and $h(x) = -2^x$. Compare the rates of change, y-intercepts, and end behaviors of each graph.

	$f(x)$	$g(x)$	$h(x)$
End Behavior:	As x increases, f(x) → ∞. As x decreases, f(x) → 0.	As x increases, f(x) → ∞. As x decreases, f(x) → 0.	As x increases, f(x) → −∞. As x decreases, f(x) → 0.
y-intercept	(0,1)	(0,5)	(0,−1)



32. Two quantities increase at exponential rates. This table shows the value of Quantity A, f(x), after x years.

Quantity A

x	0	1	2	3	4
f(x)	100.00	150.00	225.00	337.50	506.25

This function represents the value of Quantity B, $g(x)$, after x years.

$$g(x) = 50(2)^x$$

Which quantity will be greater at the end of the fourth year and by how much?

Quantity A at 4 years = 506.25

Quantity B at 4 years = $50(2)^4$ = 800

Quantity B will be 293.75 units greater than Quantity A.

33. Look at the graph. Which equation represents this graph?

A. $y = 2^{(x+1)} - 2$

B. $y = 2^{(x-1)} + 2$

C. $y = 2^{(x+2)} - 1$

D. $y = 2^{(x-2)} + 1$