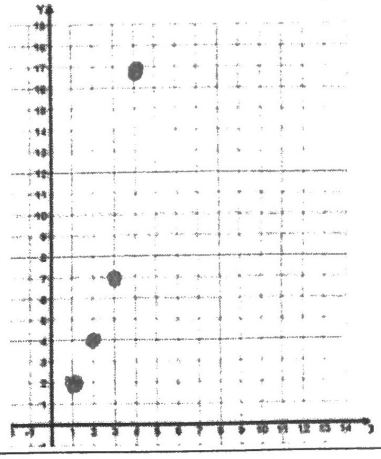
Assigned: Monday 4/23

DUE ON FRIDAY 4/27, MUST SHOW WORK FOR CREDIT

UNIT 5: COMPARING AND CONTRASTING FUNCTIONS

Name Key

Question	Answer
Construct and Compare Linear, Quadratic, and Exponential Models and Solve Problems	
1. Given the table below, determine if the table represents an exponential or linear function. Linear	<table><tr><th>x</th><th>f(x)</th></tr><tr><td>−2</td><td>5</td></tr><tr><td>−1</td><td>3</td></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>−1</td></tr><tr><td>2</td><td>−3</td></tr></table> −2, −2, −2, −2
2. Suppose you start work and earn $30,000 per year. After one year, you are given two choices for getting a raise: a) 2% per year or b) $600 plus a flat $15-per-year raise for each successive year. Which option is better? a) $y = 30000(1+.02)^t$ If $t = 5$ years: $y = 30000(1.02)^5 = \$33,122$ b) $y = 30600 + 15t$ $y = 30600 + 15(5) = \$30675$	It depends on how many years you will work at the company. Eventually the exponential function (choice a) will exceed the salary of b.
3. The swans on Elsworth Pond have been increasing in number each year. Felix has been keeping track, and so far he has counted 2, 4, 7, 17, and 33 swans each year for the past 5 years. 1st year, 2nd year, 3rd year **a. Make a scatter plot of the swan populations.** don't connect the dots **b. What type of model would be a better fit, linear or exponential? Explain your answer.** Exponential - the growth rate is NOT constant (nearly doubling each year) **c. How many swans should Felix expect next year if the trend continues? Explain your answer.** $y = 1(2)^t$ $y = (2)^6$ About 64 swans	
4. Given the sequence 7, 10, 13, 16, . . . **a. Does it appear to be linear or exponential?** **b. Determine a function to describe the sequence.** **c. What would the 20th term of the sequence be?**	a) Linear b) $a_n = 7 + (n-1)3$ $a_n = 7 + 3n - 3$ $a_n = 3n + 4$ c) $a_{20} = 3(20) + 4 = 64$

5. This table shows that the value of $f(x) = 5x^2 + 4$ is greater than the value of $g(x) = 2^x$ over the interval [0, 8]. As x increases, will the value of $f(x)$ always be greater than the value of $g(x)$? Explain how you know.

f(x) is quadratic b/c there is x^2

g(x) is exponential b/c there is b^x

Exponential function g(x) will eventually exceed quadratic function f(x) - somewhere between 8 & 9

x	f(x)	g(x)
0	$5(0)^2 + 4 = 4$	$2^0 = 1$
2	$5(2)^2 + 4 = 24$	$2^2 = 4$
4	$5(4)^2 + 4 = 84$	$2^4 = 16$
6	$5(6)^2 + 4 = 184$	$2^6 = 64$
8	$5(8)^2 + 4 = 324$	$2^8 = 256$
9	$5(9)^2+4=409$	$2^9 = 510$
10	$5(10)^2+4=504$	$2^{10}=1024$

6. How does the growth rate of the function $f(x) = 2x + 3$ compare with $g(x) = 0.5x^2 - 3$? Use a graph to explain your answer.

f(x) has a constant rate of change (linear)

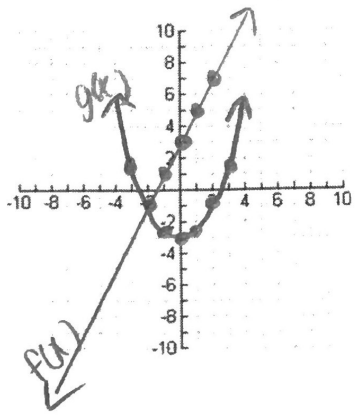
g(x) has a variable rate of change (quadratic)

f(x):

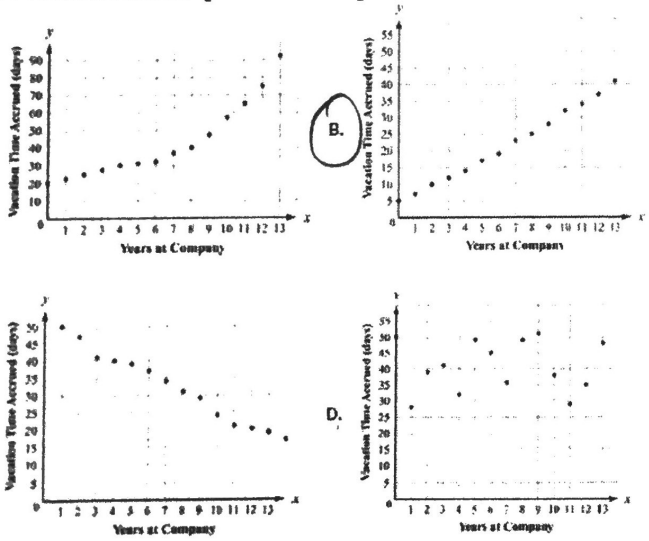
x	y
-2	-1
-1	1
0	3
1	5
2	7

g(x):

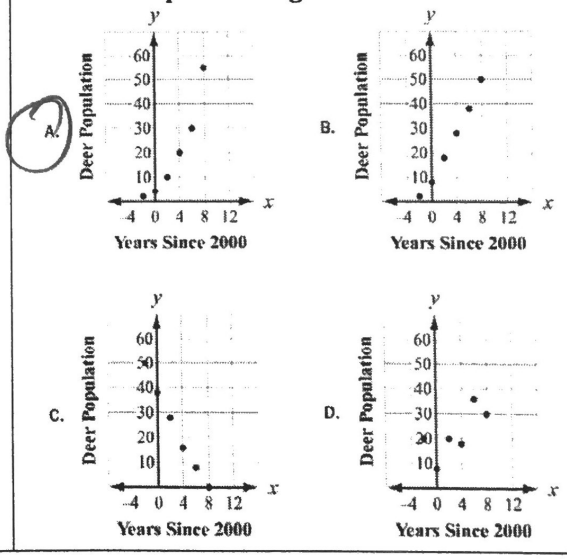
x	y
-2	-1
-1	-2.5
0	-3
1	-2.5
2	-1
3	1.5



7. Which scatter plot BEST represents a model of linear growth?



8. Which scatter plot BEST represents a model of exponential growth?



9. Which statement is true about the graphs of exponential functions?

A. The graphs of exponential functions never exceed the graphs of linear and quadratic functions.
B. The graphs of exponential functions always exceed the graphs of linear and quadratic functions.
C. The graphs of exponential functions eventually exceed the graphs of linear and quadratic functions.
D. The graphs of exponential functions eventually exceed the graphs of linear functions but not quadratic functions.

10. **Which table represents an exponential function?**

A.

x	0	1	2	3	4
y	5	6	7	8	9

B.

x	0	1	2	3	4
y	0	22	44	66	88

C.

x	0	1	2	3	4
y	5	13	21	29	37

D.

x	0	1	2	3	4
y	0	3	9	27	81

11. A table of values is shown for f(x) and g(x).

x	f(x)
0	0
1	1
2	4
3	9
4	16
5	25

x	g(x)
0	−2
1	−1
2	1
3	5
4	13
5	29

Which statement compares the graphs of $f(x)$ and $g(x)$ over the interval [0, 5]?

A. The graph of $f(x)$ always exceeds the graph of $g(x)$ over the interval [0, 5].

B. The graph of $g(x)$ always exceeds the graph of $f(x)$ over the interval [0, 5].

C. The graph of $g(x)$ exceeds the graph of $f(x)$ over the interval [0, 4], the graphs intersect at a point between 4 and 5, and then the graph of $f(x)$ exceeds the graph of $g(x)$.

D. The graph of $f(x)$ exceeds the graph of $g(x)$ over the interval [0, 4], the graphs intersect at a point between 4 and 5, and then the graph of $g(x)$ exceeds the graph of $f(x)$.

12. Which statement BEST describes the comparison of the function values for $f(x)$ and $g(x)$?

x	f(x)	g(x)	
0	0	−10	+1
1	2	−9	+3
2	4	−6	+5
3	6	−1	+7
4	8	6	+9
5	10	15	

(+2 between each pair of f(x) values)

A. The values of $f(x)$ will always exceed the values of $g(x)$.

B. The values of $g(x)$ will always exceed the values of $f(x)$.

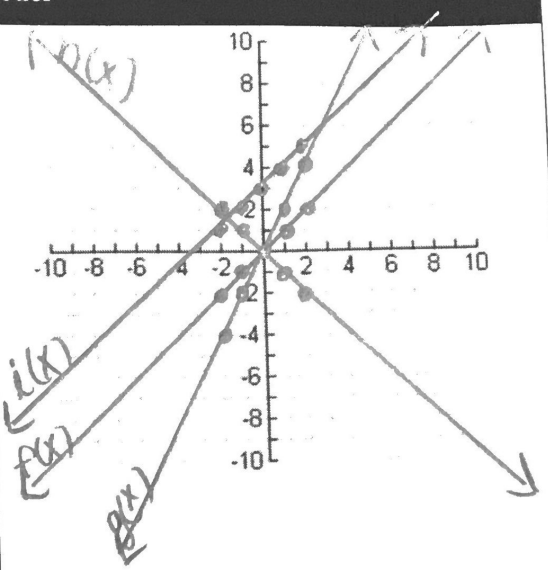
C. The values of $f(x)$ exceed the values of $g(x)$ over the interval [0, 5].

D. The values of $g(x)$ begin to exceed the values of $f(x)$ within the interval [4, 5]

Interpret Expressions for Functions in Terms of the Situation They Model

13. Consider the lines $f(x) = x$, $g(x) = 2x$, $h(x) = -x$, and $i(x) = x + 3$. Make a table and graph. Compare.

	$f(x)$	$g(x)$	$h(x)$	$i(x)$
y-intercept	(0,0)	(0,0)	(0,0)	(0,3)
Rate of Change	1	2	-1	1
Translations /Reflections	none	None (vertical stretch)	Reflect over x-axis	Up 3



f(x)

x	y
-2	-2
-1	-1
0	0
1	1
2	2

g(x)

x	y
-2	-4
-1	-2
0	0
1	2
2	4

h(x)

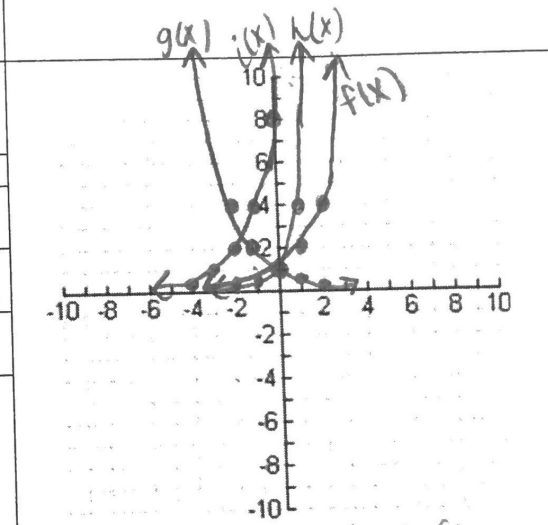
x	y
-2	2
-1	1
0	0
1	-1
2	-2

i(x)

x	y
-2	1
-1	2
0	3
1	4
2	5

14. Consider the exponential curves $f(x) = 2^x$, $g(x) = 2^{-x}$, $h(x) = 2^{2x}$, and $i(x) = 2^{x+3}$.

	$f(x)$	$g(x)$	$h(x)$	$i(x)$
y-intercept	(0,1)	(0,1)	(0,1)	(0,8)
Rate of Change	Positive	Negative	Positive	Positive
Translations /Reflections	None	Reflect over y-axis	~~horizontal stretch~~ (SKIP)	Left 3



- h(x) is f(x) shifted 3 left
- f(x) & g(x) are mirror images over y-axis

f(x)

x	y
-2	.25
-1	.5
0	1
1	2
2	4

g(x)

x	y
-2	4
-1	2
0	1
1	.5
2	.25

h(x)

x	y
-2	.0625
-1	.25
0	1
1	4
2	16

i(x)

x	y
-2	2
-1	4
0	8
1	16
2	32

15. Katherine has heard that you can estimate the outside temperature from the number of times a cricket chirps. It turns out that the warmer it is outside, the more a cricket will chirp. She has these three pieces of information:

- **A cricket chirps 76 times a minute at 56° (76, 56).** (x, y)
- **A cricket chirps 212 times per minute at 90° (212, 90).** (x, y)
- **The relationship is linear.** $y = mx + b$

Estimate the function.

$$m = \frac{90-56}{212-76} = \frac{34}{136} = \frac{1}{4}$$

$y = \frac{1}{4}x + b$

$56 = \frac{1}{4}(76) + b$

$56 = 19 + b$

$37 = b$

$$y = \frac{1}{4}x + 37$$

16. **Alice finds that her flower bulbs multiply each year. She started with just 24 tulip plants. After one year she had 72 plants. Two years later she had 120. Find a linear function to model the growth of Alice's bulbs.**

y-int → 0

$y = mx + b$

Year	0	1	2	3	4
Flower Bulbs	24	72	120	168	216

+48 +48 +48 +48

$72 - 24 = 48$

$m = 48$

$b = 24$

$\boxed{y = 48x + 24}$

17. **Suppose Alice discovers she counted wrong the second year and she actually had 216 tulip plants. She realizes the growth is not linear because the rate of change was not the same. She must use an exponential model for the growth of her tulip bulbs. Find the exponential function to model the growth.**

$y = ab^x$

0	1	2
24	72	216

×3 ×3

$\frac{72}{24} = 3$

* multiply by 3 each year

$\boxed{y = 24(3)^x}$

18. **If the parent function is $f(x) = mx + b$, what is the value of the parameter m for the line passing through the points (−2, 7) and (4, 3)?**

A. −9

B. $-\frac{3}{2}$

C. −2

D. $-\frac{2}{3}$

$m = \frac{3-7}{4-(-2)} = \frac{-4}{6}$

$= \boxed{-\frac{2}{3}}$

19. **Consider this function for cell duplication. The cells duplicate every minute.**

$f(x) = 75(2)^x$

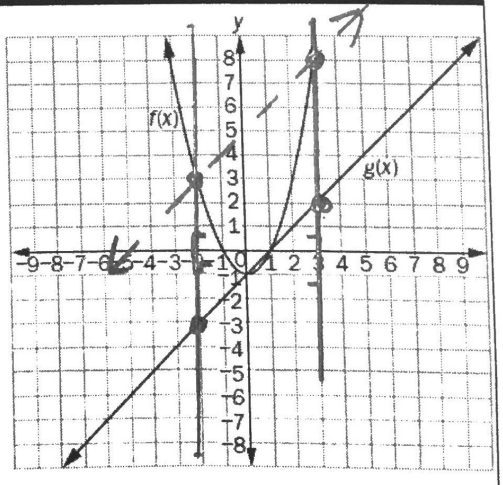
Describe the parameters of this function.

- There were 75 cells initially
- The cells double (×2) each minute

Build New Functions from Existing Functions

20. **Look at the graphs of the function $f(x) = x^2 + 1$ and $g(x) = x - 1$.**

What transformation makes $g(x) \geq f(x)$ only for the interval $-2 \leq x \leq 3$?

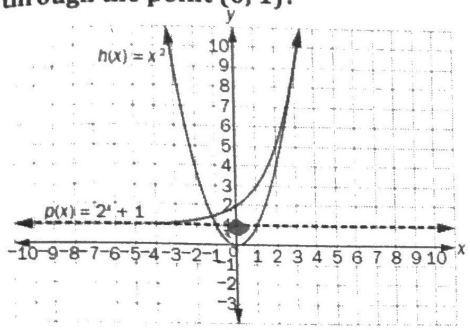


Translate g(x) up 6 units

$g(x) + 6 = (x - 1) + 6$

$= x + 5$

21. **Look at the graph of the functions h(x) and p(x). Which transformations of h(x) and p(x) translate each function so both pass through the point (0, 1)?**



A. $h(x-1) = (x-1)^2$ and $p(x+1) = 2^{(x+1)} + 1$
B. $h(x+1) = (x+1)^2$ and $p(x-1) = 2^{(x-1)} + 1$
C. $h(x) - 1 = x^2 - 1$ and $p(x) + 1 = 2^x + 1$
(D.) $h(x) + 1 = x^2 + 1$ and $p(x) - 1 = 2^x - 1$

Up 1 Down 1

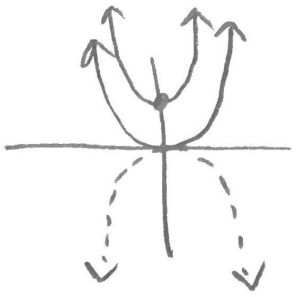
22.

Look at the functions $f(x)$ and $g(x)$.

$$f(x) = x^2$$
$$g(x) = 2^x + 3$$

Which transformation of $f(x)$ makes $f(x) < g(x)$?

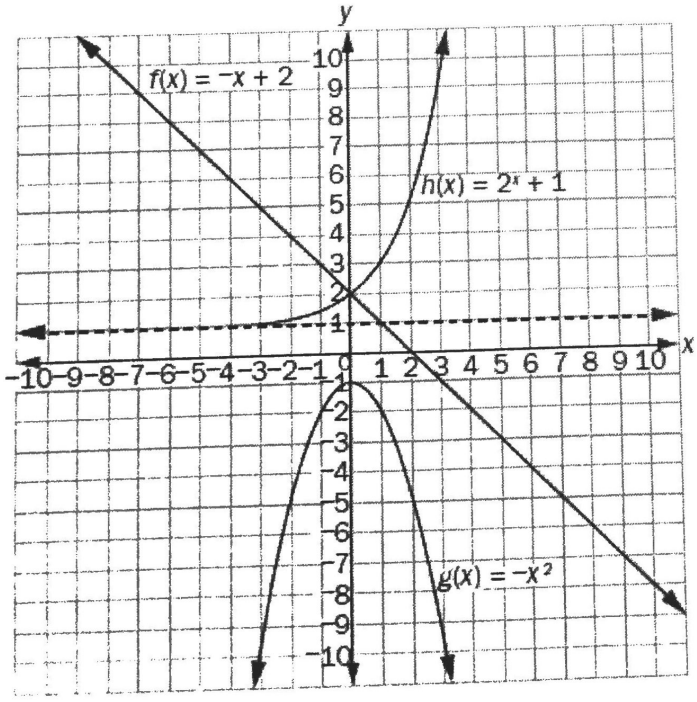
A. $f(-x)$
(B.) $-f(x)$
C. $\frac{1}{2}f(x)$
D. $2f(x)$



Understand the Concept of a Function and Use Function Notation

23. Look at the three functions and their graphs. Compare.

$f(x) = -x + 2$ $g(x) = -x^2$ $h(x) = 2^x + 1$

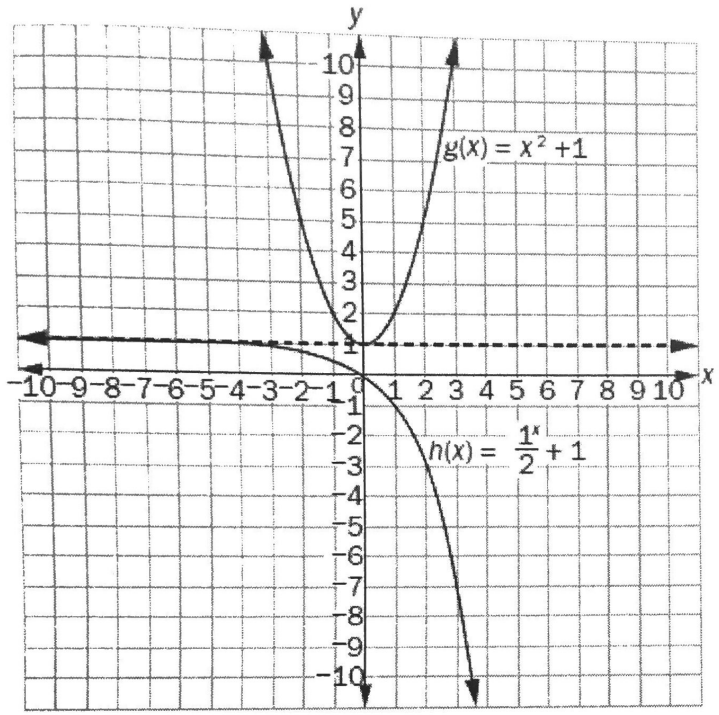


	$f(x)$	$g(x)$	$h(x)$
Domain	ℝ	ℝ	ℝ
RANGE	ℝ	$y \leq -1$	$y > 1$
ASYMPTOTE	none	none	$y = 1$

- f(x) is linear
- g(x) is quadratic, the range depends on y-value of vertex
- h(x) is exponential, the range depends on the asymptote.

24. Look at the two functions and their graphs. Compare.

$g(x) = x^2 + 1$ $\quad h(x) = \frac{1^x}{2} + 1$



	$g(x)$	$h(x)$
Domain	ℝ	ℝ
RANGE	$y \geq 1$	$y < 1$
ASYMPTOTE	none	$y = 1$
MINIMUM	(0,1)	none

Quadratic
Only quadratics have a vertex (min/max)

exponential
Only exponentials have an asymptote

25. A manufacturer keeps track of her monthly costs by using a "cost function" that assigns a total cost for a given number of manufactured items, x. The function is $C(x) = 5{,}000 + 1.3x$.

a. What is the reasonable domain of the function?
b. What is the cost of 2,000 items?

a) since x represents the number of items, it cannot be negative.
$[0, \infty)$ or $x \geq 0$

b) $C(2000) = 5000 + 1.3(2000)$
$= \$7600$

26. As the input x increases by a factor of 3, the output g(x) doubles. What type of function fits this situation?

Exponential

27. Which function is modeled in this table?

x	f(x)
1	8
2	40
3	200
4	1,000

×5 ×5 ×5 ÷5

A. $f(x) = x + 7$
B. $f(x) = 5x + 8$
} Linear

C. $f(x) = (8)^x$
D. $f(x) = \frac{8}{5}(5)^x$
} Exponential

base = 5
initial value = $\frac{8}{5}$ (when x=0)

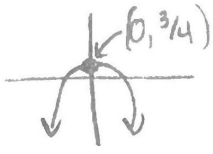
28. If $f(12) = 4(12) - 20$, which function gives $f(x)$?

A. $f(x) = 4x^2 - 20$
B. $f(x) = 4^x - 20$
C. $f(x) = 4x - 20$
D. $f(x) = 4x^2 + 12x - 20$

29.

Which function has a range of $f(x) \leq \frac{3}{4}$?

A. $f(x) = \frac{3}{4}x + 5$
B. $f(x) = -x^2 + \frac{3}{4}$
C. $f(x) = x^2 - \frac{3}{4}$
D. $f(x) = \frac{3}{4} - 5x$



Interpret Functions That Arise in Applications in Terms of the Context

30. Roger is washing cars for people in his neighborhood. He bought cleaning supplies with his own money before he began washing cars. He charges a flat fee of $15 for each car washed. Roger's total amount of profit, y, in dollars, for washing x cars can be modeled by the function $y = 15x - 40$.

a. What is the domain of this function?

b. What does the y-intercept of this function represent?

a) X represents number of cars washed. This could be zero or more. $x \geq 0$ or $[0, \infty)$

b) If he washes no cars, he loses $40, meaning he spent $40 on supplies

31. Miranda has an investment that earns 8% interest each year. She calculates that over the first 5 years, her $1,000 investment will earn an average of approximately $94 per year. At this rate, she thinks it will take more than 10 years to double her money. The graph shows the function modeling her investment, $V(t) = 1{,}000(1.08)t$, where t represents the time in years.

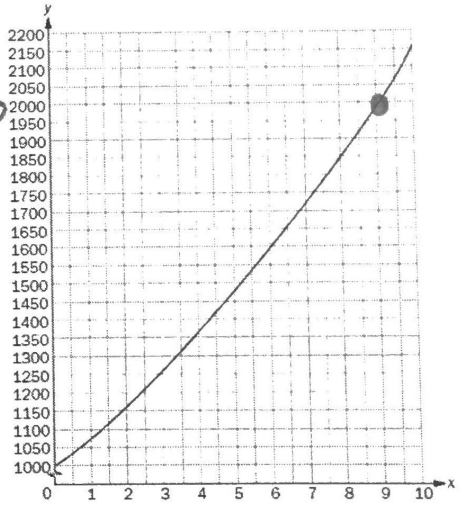
double investment = $2000

a. Approximately how many years does it actually take for Miranda to double her initial investment?

9 years

b. Explain why Miranda's estimate was incorrect.

She did not realize the growth rate was exponential, so the amount she earns will change each year.



32. A sample of 1,000 bacteria becomes infected with a virus. Each day, one-fourth of the bacteria sample dies due to the virus. A biologist studying the bacteria models the population of the bacteria with the function $P(t) = 1{,}000(0.75)^t$, where t is the time, in days. What is the range of this function in this context?

Range is the number of bacteria

A. any real number such that $t \geq 0$
B. any whole number such that $t \geq 0$
C. any real number such that $0 \leq P(t) \leq 1{,}000$
D. any whole number such that $0 \leq P(t) \leq 1{,}000$

33. The graph shows the height, y, in meters, of a rocket above sea level in terms of the time, t, in seconds, since it was launched. The rocket landed at sea level.

What does the x-intercept represent in this situation?

A. the height from which the rocket was launched
B. the time it took the rocket to return to the ground
C. the total distance the rocket flew while it was in flight
D. the time it took the rocket to reach the highest point in its flight