**Graphing Transformations (Discovery Task) – Teacher Notes**

Common Student Misconceptions

1. Students may believe that the graph of *y* = (*x*–4)3 is the graph of *y* = *x*3 shifted 4 units to the left

 (due to the subtraction symbol). Examples should be explored by hand and on a graphing calculator

 to overcome this misconception.

2. Students often confuse the shift of a function with the stretch of a function.

3. Students may also believe that even and odd functions refer to the exponent of the variable, rather

 than the sketch of the graph and the behavior of the function.

4. Additionally, students may believe that all functions have inverses and need to see counter

 examples, as well as examples in which a non–invertible function can be made into an invertible

 function by restricting the domain. For example, *f*(*x*) = *x*2 has an inverse (*f*–1(*x*)= $\sqrt{x}$ provided that

 the domain is restricted to $x\geq 0$.

***This problem set is provided for students to explore the transformations of the graph of f(x) = x2. Depending on their comfort level with technology, some students may struggle using the graphing calculator. The teacher should plan on circulating throughout the room to help students use the correct functions on the graphing calculator. For this problem set, students will learn more about the graphs of parabolas if they struggle a bit. Teachers should make sure that students actually make the graphs, because many students will be able to immediately write the equations with the given information.***

You will graph various functions and make conjectures based on the patterns you observe from the original function *y* = *x*2.

**Follow the directions below and answer the questions that follow.**

* Fill in the t–chart and sketch the parent graph ***y* =** ***x*2** below.

***y***

***x***

|  |  |
| --- | --- |
| ***x*** | ***y* = *x*2** |
| –3 | **9** |
| –2 | **4** |
| –1 | **1** |
| 0 | **0** |
| 1 | **1** |
| 2 | **4** |
| 3 | **9** |

* Now, for each set of problems below, describe what happened to the parent graph

 (*y*1 = *x*2) to get the new functions.

|  |  |
| --- | --- |
| **Equation** | **Changes to parent graph.** |
| *y*1 = *x*2*y*2 = *x*2 + 3*y*3 = *x*2 + 7 | ***Students should observe the graph shifting up*** |

1. Conjecture: The graph of *y* = *x*2 + *a* will cause the parent graph to ***shift up***.

|  |  |
| --- | --- |
| **Equation** | **Changes to parent graph.** |
| *y*1 = *x*2 *y*2 = *x*2 – 3*y*3 = *x*2 – 7 | ***Students should observe the graph shifting down*** |

1. Conjecture: The graph of *y* = *x*2 – *a* will cause the parent graph to ***shift down.***

|  |  |
| --- | --- |
| **Equation** | **Changes to parent graph.** |
| *y*1 = *x*2 *y*2 = (*x* + 3)2*y*3 = (*x* + 7)2 | ***Students should observe the graph shifting left. There will be much discussion about why there is a positive sign in the parenthesis, but the graph is shifting left.***  |

1. Conjecture: The graph of *y* = (*x* + *a*)2will cause the parent graph to ***shift left.***

|  |  |
| --- | --- |
| **Equation** | **Changes to parent graph.** |
| *y*1 = *x*2*y*2 = (*x* – 3)2*y*3 = (*x* – 7)2 | ***Students should observe the graph shifting right. There will be much discussion about why there is a negative sign in the parenthesis, but the graph is shifting right.*** |

1. Conjecture: The graph of *y* = (*x* – *a*)2 will cause the parent graph to ***shift right.***

|  |  |
| --- | --- |
| **Equation** | **Changes to parent graph.** |
| *y*1 = *x*2*y*2 = –*x*2*y*3 = –3*x*2 | ***Students will say “flip” or “turn upside down”. Make sure they use the appropriate word, reflection. For later courses, it is important to make sure they know this is a reflection about the x–axis.***  |

5. Conjecture: Multiplying the parent graph by a negative causes the parent graph to ***reflect across the x–axis.***

 ***For the following graphs, please use the descriptions “vertical stretch” (skinny)***

 ***or “vertical shrink” (fat).***

|  |  |
| --- | --- |
| **Equation** | **Changes to parent graph.** |
| *y*1 = *x*2*y*2 = 3*x*2*y*3 = 7*x*2 | ***vertical stretch***  |

6. Conjecture: Multiplying the parent graph by a number whose absolute value is greater than one causes the parent graph to ***vertical stretch***

|  |  |
| --- | --- |
| **Equation** | **Changes to parent graph.** |
| *y*1 = *x*2*y*2 = ½ *x*2*y*3 = ¼ *x*2 | ***vertical shrink*** |

7. Conjecture: Multiplying the parent graph by a number whose absolute value is between zero and one causes the parent graph to ***vertical shrink.***

**Based on your conjectures above, sketch the graphs without using your graphing calculator.**

8. *y* = (*x* + 3)2 – 4 9. *y* = –*x*2 + 5

***y***

***x***

***y***

***x***

**Now, go back and graph these on your graphing calculator and see if you were correct.**

**Were you?**

**Based on your conjectures, write the equations for the following transformations to *y* = *x*2.**

10.Translated 6 units up 11. Translated 2 units right

 ***y = x2 + 6 y = (x–2)2***

12.Stretched vertically by a factor of 3 13. Reflected over the x–axis, 2 units left and down 5 units

 ***y = 3x2 y = –(x+2)2–5***

***More problems can be added as an extension.***