

Name: _____
Day 19 Algebra 1

Date: _____ Period: _____
Arithmetic and Geometric Sequences: Explicit Rule

A Sequence is a set of numbers, called the terms of the sequence, in a specific order. Look for a pattern in the information given in the table below representing a women's crew team.

Distance (meters)	400	800	1200	1600	2000
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What pattern do you notice in the Distances? Adding 400

This is an arithmetic sequence, since the difference between successive terms is constant (adding the same number each time). The difference between the terms is called the Common difference, d . To find the common difference, you always take the previous term and subtract it from the successive term.

$2^{nd} - 1^{st}$

Example 1: Find the next three terms in each arithmetic sequence by using the common difference.

a) $-26, -22, -18, -14, \dots$
 $d = \underline{4}$ $-22 - (-26) = 4$ ^{adds}

Next 3 terms: $-10, -6, -2$

b) $15, 9, 3, -3, \dots$ $d = 9 - 15 = -6$
 $3 - 9 = -6$
 $d = \underline{-6}$

Next 3 terms: $-9, -15, -21$

It is important to check that the common difference works throughout the entire sequence!

Amount of Flu Patients	4	8	16	32	64
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What pattern do you notice in the amount of flu patients? Multiplying by 2

In a geometric sequence, the first term is nonzero and each term after the first is found by multiplying the previous term by a constant, r , called the Common ratio. The common ratio can be found by dividing any term by its previous term.

$2^{nd} \div 1^{st}$

Example 2: Find Terms of Geometric Sequences

Find the next three terms in each geometric sequence by using the common ratio.

a) $1, -4, 16, -64, \dots$
 $r = \underline{-4}$ $\frac{-4}{1} = -4$

Next 3 terms: $256, -1024, 4096$

b) $9, 3, 1, \frac{1}{3}, \dots$ $d = 3 - 9 = -6$
 $1 - 3 = -2$
 $r = \underline{\frac{1}{3}}$ $\frac{1}{3} \div 3 = \frac{1}{9}$
dividing by 3 = multiplying by 1/3

Next 3 terms: $\frac{1}{9}, \frac{1}{27}, \frac{1}{81}$

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c) -3, 15, -75, 375, ...

$$r = \frac{15}{-3} = -5$$

Next 3 terms: -1875, 9375, -46875

d) 24, 36, 54, 81, ...

$$r = \frac{36}{24} = \frac{3}{2} = 1.5$$

Next 3 terms: 121.5, 182.25, 273.375

Example 3: Identify Geometric and Arithmetic Sequences

Determine whether each sequence is *arithmetic*, *geometric*, or *neither*. Explain.

a) 256, 128, 64, 32, ...

$$\frac{128}{256} = .5 \quad \frac{64}{128} = .5 \quad \frac{32}{64} = .5 \quad r = \frac{1}{2}$$

Geometric

c) -20, -15, -10, -5, ...

Arithmetic $d=5$

e) 1, 3, 9, 27, ...

Geometric
 $r=3$

b) 4, 9, 12, 18, ...

$$\frac{9}{4} = 2.25 \quad \frac{12}{9} = 1.33$$

Neither

d) 2, 8, 14, 22, ...

$\begin{matrix} \downarrow & \downarrow & \downarrow \\ 16 & +6 & +8 \end{matrix}$ Neither

f) 94, 79, 64, 49, ...

Arithmetic $d=-15$

Arithmetic Key Concept: The n th term of an arithmetic sequence with the first term a_1 and common difference, d , is given by the explicit formula:

$$a_n = a_1 + (n - 1)d, \text{ where } n \text{ is a positive integer.}$$

Geometric Key Concept: The n th term of a geometric sequence with the first term, a_1 , and common ratio, r , is given by the following formula:

$$a_n = a_1 \cdot (r)^{n-1}, \text{ where } n \text{ is any positive integer and } a_1, r \neq 0.$$

Example 4: Write an equation for the n th term of the sequence -12, -8, -4, 0, ...

a) Is the sequence arithmetic or geometric? Find the common difference/ratio.

Arithmetic $d=4$

b) Substitute a_1 and d in the formula $a_n = a_1 + (n - 1)d$ and simplify

$$a_1 = -12$$

$$d = 4$$

$$a_n = -12 + 4(n-1) = -12 + 4n - 4$$

$$a_n = 4n - 16$$

c) Find a_9 , the 9th term of the sequence using your formula from number 2.

$$a_9 = 4(9) - 16 =$$

$$36 - 16 =$$

20

← $9^{\text{th}} \text{ term}$

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Example 5: Write an equation for the n th term of the sequence $-6, 12, -24, 48, \dots$. Then, find the 9th term.

Step 1: Determine if the sequence is arithmetic or geometric. Find the common difference or ratio.

Geometric $r = -2$

Step 2: Substitute a_1 and the common ratio into the formula.

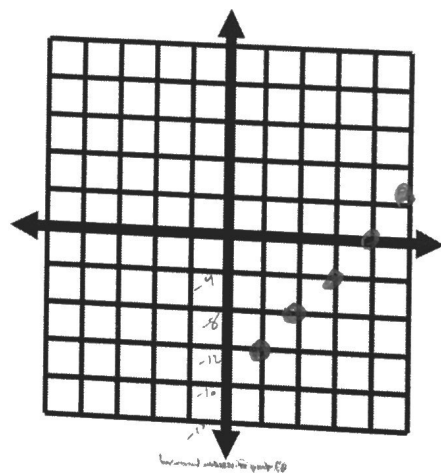
$a_1 = -6$
 $r = -2$
 $a_n = a_1(r)^{n-1}$
 $a_n = -6(-2)^{n-1}$

Step 3: Find the a_8 , the 8th term of the sequence by substituting 8 in for n .

$a_8 = -6(-2)^{8-1}$
 $= -6(-2)^7 = -6(-128) = \boxed{768 = a_8}$ 8th term

Example 6: Graph the first five terms of the sequence shown in the table below.

n	$a_n = 4n - 16$	a_n	Coordinate Point (n, a_n)
1	$4(1) - 16$	-12	$(1, -12)$
2	$4(2) - 16$	-8	$(2, -8)$
3	$4(3) - 16$	-4	$(3, -4)$
4	$4(4) - 16$	0	$(4, 0)$
5	$4(5) - 16$	4	$(5, 4)$



a) Is the sequence discrete or continuous? How are graphs of arithmetic sequences and linear functions similar? Different?

Discrete (don't connect the dots)

Arithmetic sequences: points make a straight line, but discrete.

b) What is the common difference?

$d = 4$ (slope = 4)

c) Find the 32nd term of the sequence.

$a_{32} = 4(32) - 16 = 128 - 16 = \boxed{112}$ ← 32nd term

d) Which term of the sequence is 96?

$4n - 16 = 96$
 $\quad \quad \quad +16 \quad \quad +16$
 $4n = 112$
 $\quad \quad \quad \div 4 \quad \quad \div 4$
 $n = \boxed{28^{\text{th}} \text{ term}}$

You try:

- 1) Write an equation for the nth term of the geometric sequence 96, 48, 24, 12, ... Then find the 10th term of the sequence.

$$r = \frac{48}{96} = .5 \quad a_1 = 96$$

$$a_n = 96(.5)^{n-1}$$

10th term:

$$a_{10} = 96(.5)^9$$

$$a_{10} = -1875$$

- 2) Write an equation for the nth term of the arithmetic sequence 3, -10, -23, -36, Find the 15th term of the sequence.

$$d = -13 \quad a_1 = 3$$

$$a_n = a_1 + (n-1)d$$

$$a_n = 3 + -13(n-1) = 3 - 13n + 13$$

$$a_n = -13n + 16$$

$$a_{15} = -13(15) + 16$$

$$a_{15} = -179$$

- 3) Write an equation for the nth term of the sequence -9, 27, -81, ... Then find the 15th term of the sequence.

Geometric $r = -3 \quad a_1 = -9$

$$a_n = -9(-3)^{n-1}$$

$$a_{15} = -9(-3)^{14} = -43,046,721$$

- 4) The graph of an arithmetic function is linear (choose one: linear/quadratic/exponential).

The graph of a geometric function is exponential (choose one: linear/quadratic/exponential).

- 5) Challenge: Find the 8th term of a geometric sequence for which $a_3 = 81$ and the common ratio is 3.

$$\frac{9}{-}, \frac{27}{-}, \frac{81}{-}$$

$$a_1 = 9$$

$$r = 3$$

$$a_n = 9(3)^{n-1}$$

$$a_8 = 9(3)^7 = 19,683$$