

Exponential Growth and Decay Models**Basic Exponential Growth Model:**

growth: $y = a(1+r)^t$

Basic Exponential Decay Model:

decay: $y = a(1-r)^t$

→ What do the variables represent?

 $y =$ output value $r =$ growth/decay rate $a =$ initial value $t =$ time→ The value $(1+r)$ is considered the growth factor, and the value $(1-r)$ is considered the decay factor. It's the portion you have left after each decay.→ When you answer word problems, don't forget to include units. Your answer should never be just a number. Write it in a complete sentence.

→ Keywords to look for in word problems:

Word	Base
Doubles	2
Triples	3
Quadruples	4
Halves	$\frac{1}{2}$
Quartered	$\frac{1}{4}$

Example 1: In January 1993, there were approximately 1.4 million internet hosts. During the next five years, the number of hosts increased by about 90% each year.1. Write a model giving the number y of hosts (in millions) t years after 1993.

$$y = 1.4(1 + 0.90)^t \rightarrow y = 1.4(1.9)^t$$

2. How many hosts were there in 1997? $1997 - 1993 = 4$ years

$$y = 1.4(1.9)^4 = 18.2 \text{ million internet hosts in 1997.}$$

Example 2: Bob bought a new car for \$21,100. The value y of cars decreases by about 18% per year.3. Write a model giving the value of Bob's car after t years.

$$y = 21100(1 - 0.18)^t = 21100(0.82)^t$$

4. How much is his car worth if he has owned it for 10 years?

$$y = 21100(0.82)^{10} = \$2,900.15$$

The car is worth \$2,900.15 after 10 years

Example 3: Georgia has 36.8 million acres of land area, 66% of which is forested (as of 2003). As more land is developed for subdivisions, shopping, and business complexes, the amount of land that is forested decreases by about 0.4% each year. 0.004

6. How many acres of land in Georgia were forested in 2003?

$$(36.8 \text{ million acres})(0.66) = 24.288 \text{ million acres}$$

7. Write a model that gives the number of acres (in millions) of land that is forested in Georgia after t years.

$$y = 24.288(1 - 0.004)^t = 24.288(0.996)^t$$

8. Use your model to predict the number of acres of land in Georgia that is still forested in 2025.

$$y = 24.288(0.996)^{22} = 22.24 \text{ million acres}$$

$$\begin{array}{r} 2025 \\ - 2003 \\ \hline 22 \end{array}$$

9. What percent of Georgia's land will still be forested in 2025?

22.24 is what % of 36.8 mil

$$22.24 \div 36.8 = 0.6043 \rightarrow \boxed{60.43\%}$$

Example 4: In 2000, the cost of tuition at a local university was \$4300. During the next 10 years, tuition rose by 6% per year.

a. Write a model giving the cost of tuition t years after 2000.

$$y = 4300(1 + 0.06)^t = 4300(1.06)^t$$

b. Find the cost of tuition in 2009. $2009 - 2000 = 9$

$$y = 4300(1.06)^9 = \$7,264.76 \text{ cost of tuition in 2009.}$$

Example 5: A population of termites doubles every day. Right now, the population of termites in your garage is 250.

a. Write a model representing the population of termites after t days.

$$y = 250(2)^t$$

b. How many termites will be present after 18 days?

$$y = 250(2)^{18} = 65,536,000 \text{ termites}$$

Example 6: Bacteria can multiply at an alarming rate when each bacteria splits into two new cells, thus doubling. If we start with only one bacteria which can double every hour, how many bacteria will we have by the end of one day? $1 \text{ day} = 24 \text{ hours}$

$$y = 1(2)^t$$

$$y = 1(2)^{24} = 16,777,216 \text{ bacteria at the end of 1 day}$$

Example 7: Each year the local country club sponsors a tennis tournament. Play starts with 128 participants. During each round, half of the players are eliminated. How many players remain after 5 rounds?

$$y = 128\left(\frac{1}{2}\right)^t$$

$$y = 128\left(\frac{1}{2}\right)^5 = \boxed{4 \text{ players}}$$