Key

Day 09 Notes - Graphing Quadratic Functions and Switching Forms

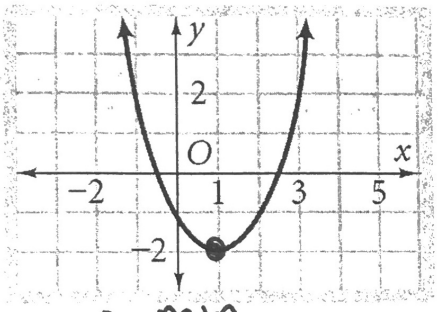
Standard Form!

A. Intro to Graphs of Quadratic Equations: $y = ax^2 + bx + c$

- A Quadratic is a function that can be written in the form $y = ax^2 + bx + c$ where a, b, and c are real numbers and $a \neq 0$. Ex: $y = 5x^2$ $y = -2x^2 + 7$ $y = x^2 - x - 3$
- The graph of a quadratic function is a U-shaped curve called a parabola. The maximum or minimum point is called the vertex

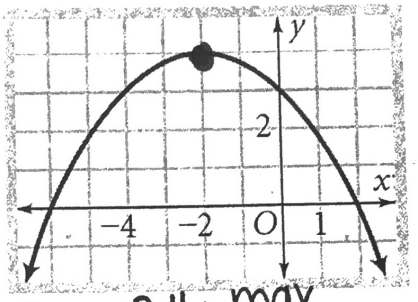
Identify the vertex of each graph; identify whether it is a minimum or a maximum.

1.)



Vertex: (1, -2) min.

2.)



Vertex: (-2, 4) max.

B. Key Features of a Parabola: $y = ax^2 + bx + c$

- **Direction of Opening:** When $a > 0$, the parabola opens up:
  When $a < 0$, the parabola opens down:
- **Width:** When $|a| < 1$, the parabola is wider than $y = x^2$
  When $|a| = 1$, the parabola is the same width as $y = x^2$
  When $|a| > 1$, the parabola is narrower than $y = x^2$
- **Vertex:** The highest or lowest point of the parabola is called the vertex, which is on the axis of symmetry. To find the vertex, plug in $x = \frac{-b}{2a}$ and solve for y. This yields a point $(\frac{-b}{2a}, y)$
- **Axis of symmetry:** This is a vertical line passing through the vertex. Its equation is: $x = \frac{-b}{2a}$
- ***x*-intercepts:** are the 0, 1, or 2 points where the parabola crosses the *x*-axis. Plug in $y = 0$ and solve for x. (factor!)
- ***y*-intercept:** is the point where the parabola crosses the *y*-axis. Plug in $x = 0$ and solve for y.

Without graphing the quadratic functions, complete the requested information:

5.) $f(x) = 3x^2 - 7x + 1$

What is the direction of opening? up
Is the vertex a max or min? min
Wider or narrower than $y = x^2$? narrower

6.) $g(x) = -\frac{5}{4}x^2 + x - 3$

What is the direction of opening? down
Is the vertex a max or min? max.
Wider or narrower than $y = x^2$? narrower

7.) $y = \frac{2}{3}x^2 - 11$

What is the direction of opening? up
Is the vertex a max or min? min
Wider or narrower than $y = x^2$? wider

8.) $y = -0.6x^2 + 4.3x - 9.1$

What is the direction of opening? down
Is the vertex a max or min? max
Wider or narrower than $y = x^2$? wider

Another useful form of the quadratic function is the vertex form: $y = a(x-h)^2 + k$.

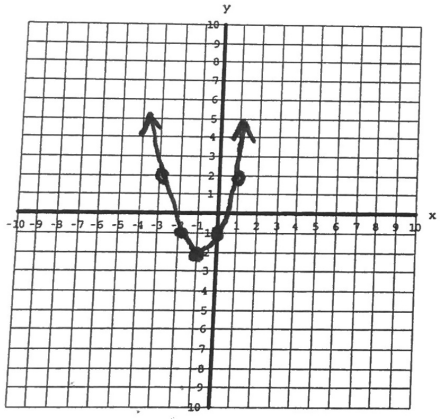
GRAPH OF VERTEX FORM $y = a(x - h)^2 + k$

The graph of $y = a(x - h)^2 + k$ is the parabola $y = ax^2$ translated left/right h units and up/down k units.

- The vertex is (h, k).
- The axis of symmetry is $x = h$.
- The graph opens up if a $>$ 0 and down if a $<$ 0.

Find the vertex of each parabola and graph.

9.) $y = (x+1)^2 - 2$



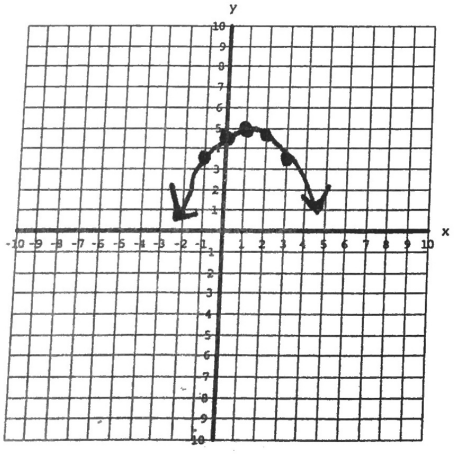
X	Y
-3	2
-2	-1
-1	-2
0	-1
1	2

mirrors around vertex!

Put vertex in middle of table!

Vertex: (-1, -2)

10.) $y = -\frac{1}{3}(x-1)^2 + 5$



X	Y
-1	3.67
0	4.67
1	5
2	4.67
3	3.67

Vertex: (1, 5)

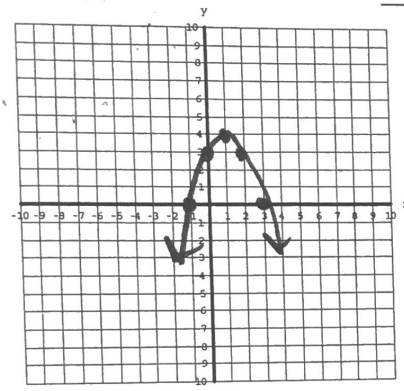
C. Graphing in STANDARD FORM ($y = ax^2 + bx + c$): we need to find the vertex first.

Vertex	Graphing
- list $a =$ ____, $b =$ ____, $c =$ ____ - find $x = \frac{-b}{2a}$ - plug this x-value into the function (table) - this point $(\frac{-b}{2a}, y)$ is the vertex of the parabola	- put the vertex you found in the center of your x-y chart. - choose 2 x-values less than and 2 x-values more than your vertex. - plug in these x values to get 4 more points. - graph all 5 points

Find the vertex of each parabola. Graph the function and find the requested information

#a stays the same!

11.) $f(x) = -x^2 + 2x + 3$ $a = -1$, $b = 2$, $c = 3$



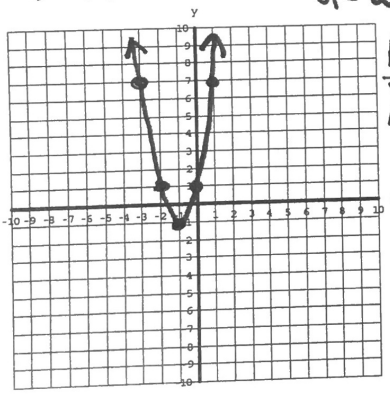
AOS: $x = \frac{-b}{2a}$ $x = \frac{-2}{2(-1)}$ $x = 1$

Vertex: $-(1)^2 + 2(1) + 3 = y$ $-1 + 2 + 3 = y$ $y = 4$

Vertex: (1, 4)
Vertex Form: $y = -(x-1)^2 + 4$
Max or min? max
Direction of opening? down
Axis of symmetry: $x = 1$
Compare to the graph of $y = x^2$
Reflect X-axis
Right 1, Down 4

x	y
-1	0
0	3
1	4
2	3
3	0

12.) $h(x) = 2x^2 + 4x + 1$ $a = 2$ $b = 4$ $c = 1$



AOS: $x = \frac{-4}{2(2)}$ $x = \frac{-4}{4}$ $x = -1$

Vertex: $y = 2(-1)^2 + 4(-1) + 1$ $y = 2 - 4 + 1$ $y = -1$

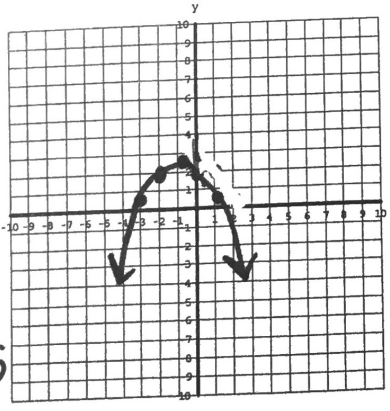
Vertex: (-1, -1)
Vertex Form: $y = 2(x+1)^2 - 1$
Max or min? min
Direction of opening? up
Axis of symmetry: $x = -1$
Compare to the graph of y = x2
Vertical Stretch by 2,
Left 1, Down 1

x	y
-3	7
-2	1
-1	-1
0	1
1	7

13.) $k(x) = 2 - x - \frac{1}{2}x^2 \rightarrow -\frac{1}{2}x^2 - x + 2$

$a = -\frac{1}{2}$ $b = -1$ $c = 2$

x	y
-3	0.5
-2	2
-1	2.5
0	2
1	0.5



A.O.S $x = \frac{-b}{2a}$ $x = \frac{1}{2(-\frac{1}{2})}$ $x = -1$

vertex $y = -\frac{1}{2}(-1)^2 - (-1) + 2$ $-\frac{1}{2} + 1 + 2 = 2.5$

Vertex: (-1, 2.5)
Vertex Form: $y = -\frac{1}{2}(x+1)^2 + 2.5$
Max or min? max
Direction of opening? Down
Axis of symmetry: $x = -1$
Compare to the graph of y = x2
Reflect X-axis, Vertical Shrink by 1/2,
Left 1, Up 2.5

14.) State whether the function $y = -3x^2 + 12x - 6$ has a minimum value or a maximum value. Then find the minimum or maximum value.

Opens down → max

$x = \frac{-b}{2a}$

$x = \frac{-12}{2(-3)} = \frac{-12}{-6} = 2$

plug in: $y = -3(2)^2 + 12(2) - 6$

$y = -3(4) + 24 - 6$

$y = -12 + 24 - 6$

$y = 6$

Vertex: (2, 6)

15.) Find the vertex of $y = \frac{1}{2}x^2 + 5x - 7$. State whether it is a minimum or maximum. Find that minimum or maximum value.

Opens up → min

$x = \frac{-b}{2a}$

$x = \frac{-5}{2(\frac{1}{2})}$

$x = \frac{-5}{1}$

$x = -5$

$y = \frac{1}{2}(-5)^2 + 5(-5) - 7$

$y = \frac{1}{2}(25) - 25 - 7$

$y = 12.5 - 25 - 7$

$y = -19.5$

Vertex: (-5, -19.5)