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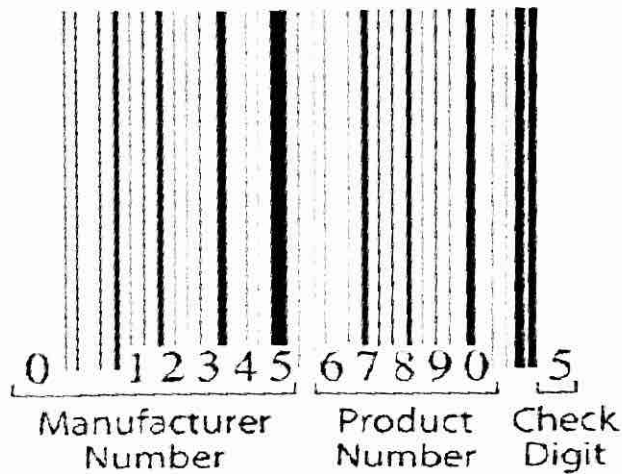
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## Analyzing Numerical Data: Validating Identification Numbers

### 1.D Student Activity Sheet 12: Universal Product Codes

Identification numbers are present everywhere in society. Today's identification numbers are more sophisticated than those introduced years earlier (for example, Social Security numbers). Today's numbers have a check digit to partially ensure that they have been correctly scanned or entered into a computer.

Universal Product Codes (UPCs), typically in the form of barcodes, identify retail products.



The 12-digit UPC barcode consists of three parts:

- manufacturer number,
- product number, and
- check digit.

For example, the manufacturer number for the Dr. Pepper Company is 078000 and appears in the first six digits of all of the company's product UPC barcodes. GS1, formerly the Uniform Code Council, issues a company this six-digit number. Every item sold by a company requires a different five-digit product number. This includes specific products, their different sizes, their array of colors, their variety of flavors, and other distinguishing features. The last number is the **check digit**, which guards against entry errors and fraud. The check digit in a UPC number (that is, the twelfth digit) is determined in the following manner:

- Multiply the first digit by 3.
- Add the second digit.
- Multiply the third digit by 3.
- Add the fourth digit.
- Continue this alternating process for the Digits 5 to 12.

multiple of 10

The check digit is chosen so that the calculation described previously totals a number whose final digit is 0. In the UPC number  $a_1a_2a_3a_4a_5a_6a_7a_8a_9a_{10}a_{11}d$ , the check digit is  $d$ , for which the sum

$3a_1 + a_2 + 3a_3 + a_4 + 3a_5 + a_6 + 3a_7 + a_8 + 3a_9 + a_{10} + 3a_{11} + d$  ends in 0. In this weighted sum, the weights are: {3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1}.

When entering a code number, the single-digit error is most common (for example, keying in 8 instead of 3). Another common error is the transposition error, where the order of two adjacent digits is reversed (for example, writing 83 instead of 38). Systems have been established to detect and correct (when possible) these and other errors almost immediately.

1. Show that 0-58200-48826-5 is a valid UPC number.

$$3(0+8+0+4+8+0) + 5+2+0+8+2+5 = 3(26) + 22 = 100 \checkmark$$

valid UPC ends in "zero"

2. Show that 0-52200-48826-5 is an invalid UPC number.

$$3(0+2+0+4+8+0) + (5+2+0+8+2+5)$$

$$= 3(20) + 22 = 60 + 22 = 82 \text{ not a valid UPC \# since it ends in "2"}$$

If someone made a single-digit error when entering this invalid number, can you tell which digit is incorrect? Why or why not?

NO. any single digit can be increased or decreased by an amount that causes the sum of the resulting 12 digits to end in "0."

Change one digit in this invalid number so the resulting number is valid.

- change the two (one of them) to a "0!"
  - change the second "0" to a 6
- \* answers will vary

3. Determine the check digit ( $d$ ) for the UPC number 38137009213d.

$$3(3+1+7+0+2+3) + (8+3+0+9+1+d)$$

$$= 3(16) + 21 + d = 48 + 21 + d = 69 + d \Rightarrow \boxed{d=1} \text{ so that the total is } 70$$

4. Suppose you entered 8 instead of 9 when recording the UPC number 1 55210 02149 6 (divisible by 10)

①  $3(1+5+1+0+1+9) + (5+2+0+2+4+6) = 3(17) + 19 = 70$

②  $3(1+5+1+0+1+8) + (5+2+0+2+4+6) = 3(16) + 19 = 67$

Explain why the UPC method will detect this error.

OR the difference is  $3(9) - 3(8) = 3$  | the difference is  $70 - 67 = 3$ .

thus, the sum of the weighted sum w/ the invalid # can't end in "0"

Do you think the UPC method will detect all such single-digit errors? Either give several examples of numbers with an error that will not be detected or explain why you think all such single-digit errors will be detected.

see attached!!

- \* 4. Do you think the UPC method will detect all such single-digit errors? Either give several examples of numbers with an error that will not be detected or explain why you think all such single-digit errors will be detected.

Yes. To show that all single-digit errors are detected, suppose Digit  $b$  is recorded instead of the correct Digit  $a$  in an even-numbered position in a UPC number. By the UPC method, you have the following:

$3a_1 + a_2 + 3a_3 + a_4 + \dots + d$  is a multiple of 10.

$3a_1 + a_2 + 3a_3 + b_4 + \dots + d$  is a multiple of 10.

Suppose an error was not detected. Then the difference between these two numbers,  $a - b$ , is also a multiple of 10, which is not possible since  $a$  and  $b$  are different digits. Thus, the UPC method will detect the error.

Now suppose Digit  $b$  is recorded instead of the correct Digit  $a$  in an odd-numbered position in a UPC number. By the UPC method, you have the following:

$3a_1 + a_2 + 3a_3 + 3a_4 + \dots + d$  is a multiple of 10.

$3a_1 + a_2 + 3a_3 + 3b_4 + \dots + d$  is a multiple of 10.

Suppose an error was not detected. Then the difference between these two numbers,  $3a - 3b = 3(a - b)$ , is also a multiple of 10, which means that  $a - b$  is a multiple of 10. Again, this is not possible since  $a$  and  $b$  are different digits. Thus, the UPC method will detect the error.