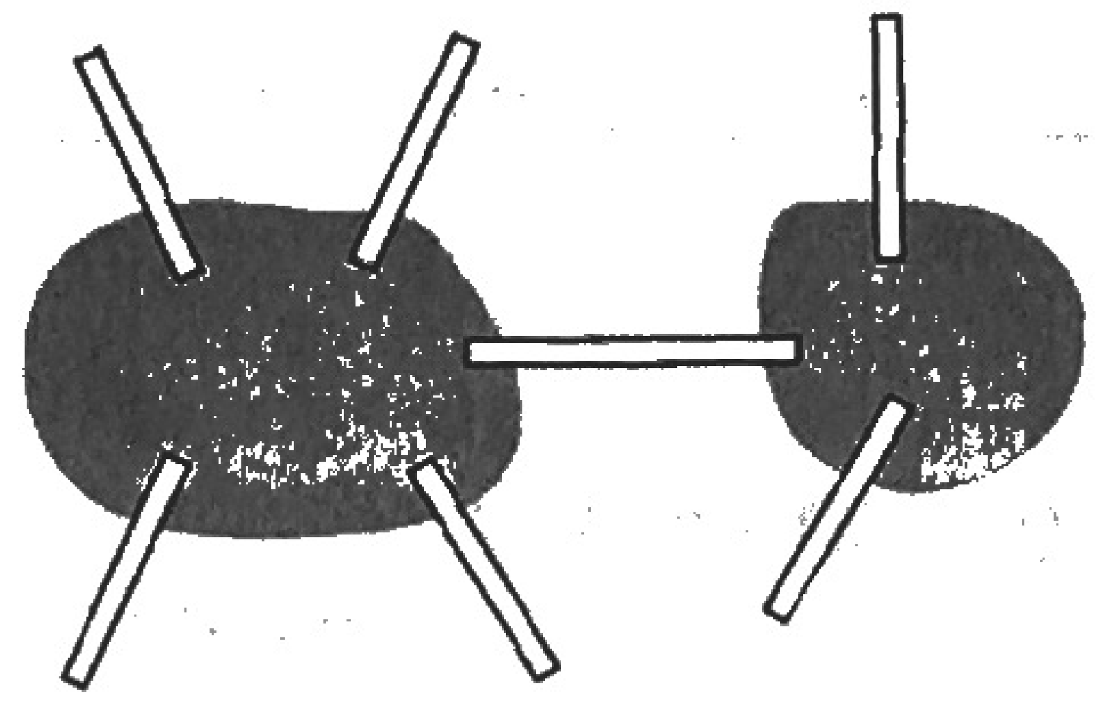
Name: KEY Date: ______________ Period: ________

Unit 7: SAS 1 Networks and Graphs: Circuits, Paths, and Graph Structures

The Königsberg Bridge Problem

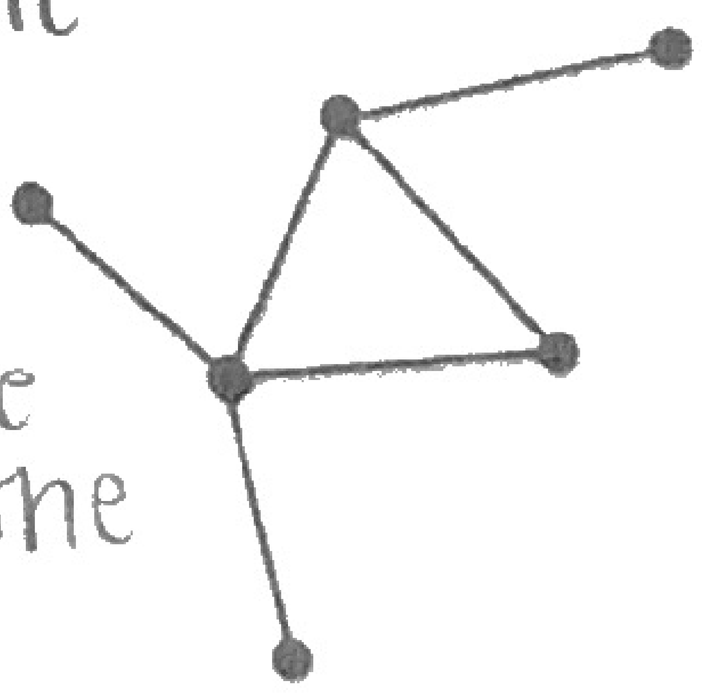
The following figure shows the rivers and bridges of Königsberg. Residents of the city occupied themselves by trying to find a walking path through the city that began and ended at the same place and crossed every bridge exactly once.



1. If you were a resident of Königsberg, where would you start your walk and what path would you choose? NO path exists!

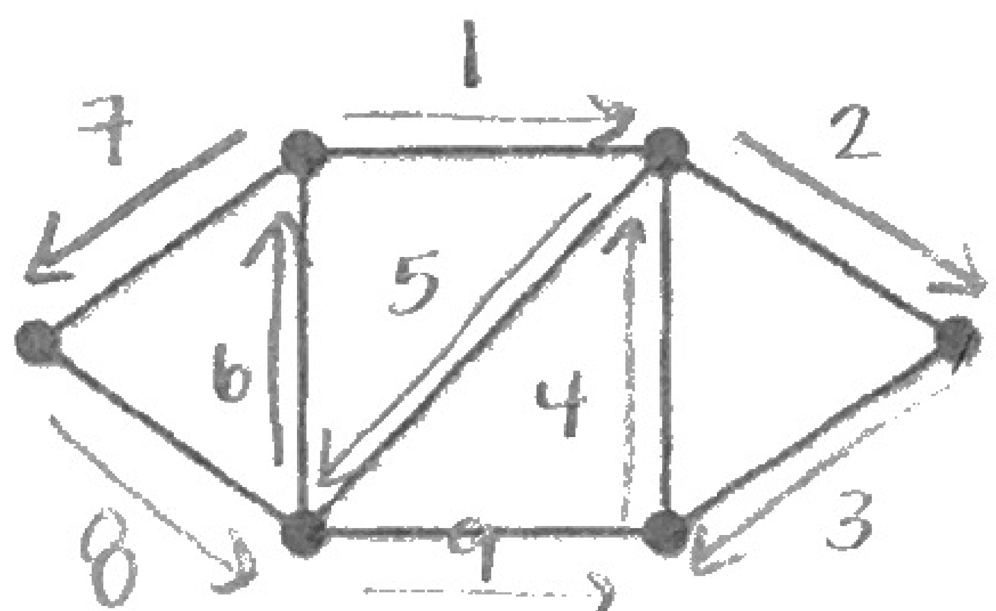
2. What about when you visit the Eastern and Western wildflower gardens that have fabulous sculptures in addition to beautiful flowers along the walkways. You want to see each display without backtracking (seeing something you have already seen). Where would you start your walk and what path would you choose?

Not possible without backtracking because of the 3 exhibits on the exterior



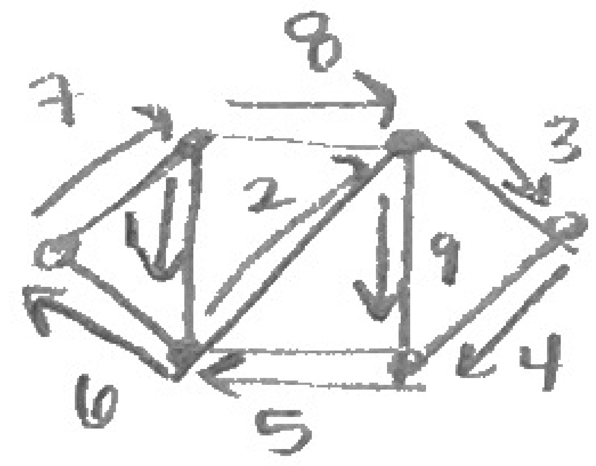
Western garden

option 1



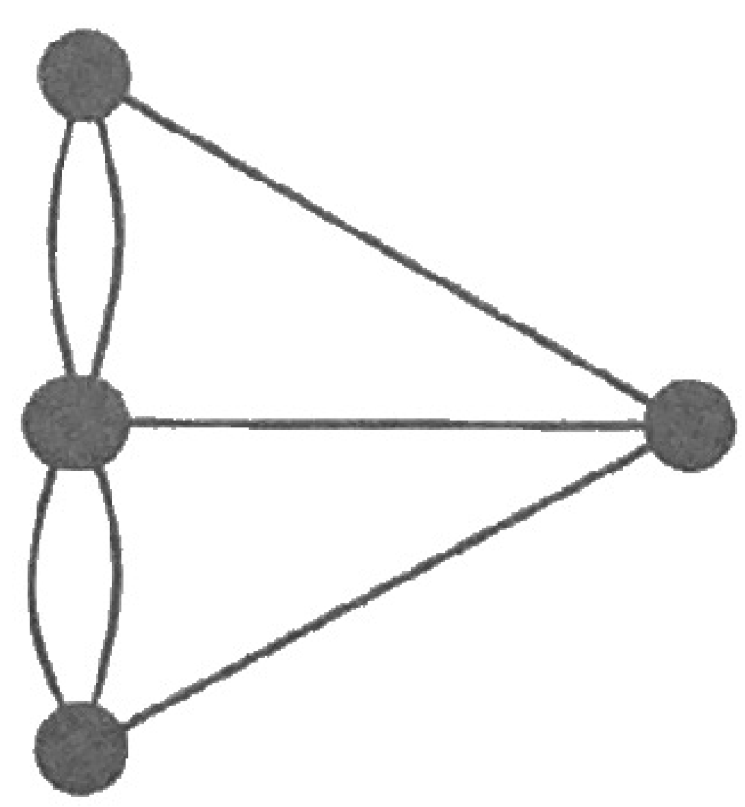
Eastern garden

option 2



*several options

3. When Leonhard Euler, a famous mathematician, turned his attention to the Bridge problem; his first step was to model the bridges of Königsberg with a simple graph. The points, or vertices, represented land and the edges represented the bridges connecting them. Euler's map of Königsberg, while much simpler, conveyed all the necessary information about which parts of land were connected by which bridges. It looked something like the following:



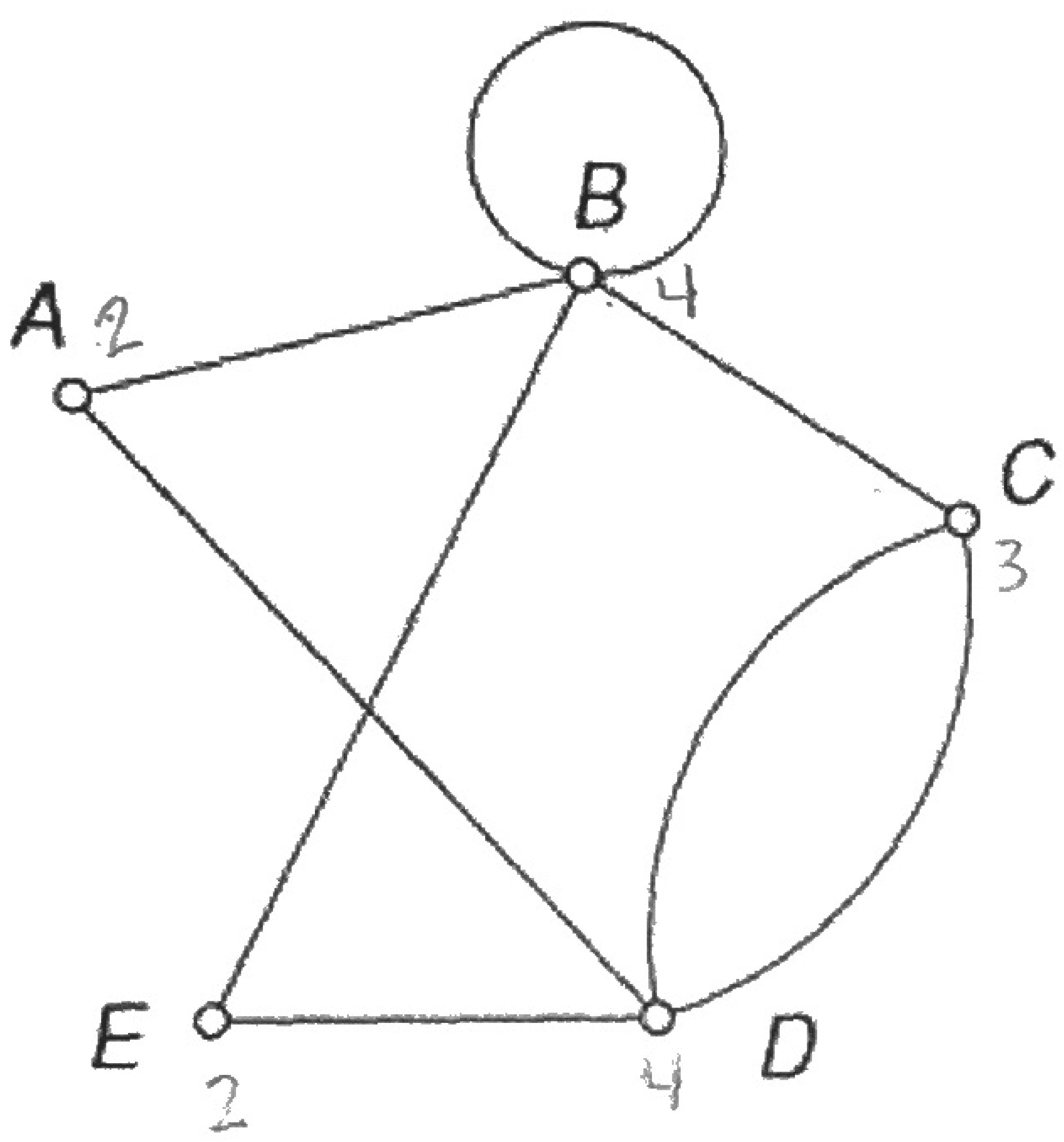
*at least 1 odd vertex – no Euler circuit *all even – at least 1 circuit

The Bridge problem is now stated: Given a graph, find a path through the vertices (points) that uses **every edge exactly once**. Such a path is called an **Euler path**. If an Euler path begins and ends at the same vertex, it is called an **Euler circuit**. >2 odd – no Euler path

Euler Path	A path that uses every edge of a graph EXACTLY ONCE.	An Euler Path STARTS and ENDS at **DIFFERENT VERTICES**.
Euler Circuit	A Circuit that uses every edge of a graph EXACTLY ONCE.	An Euler Circuit STARTS and ENDS at the **SAME VERTEX**.

Example 1: For the following diagram, come up with two Euler Paths and one Euler Circuit. There are many different possibilities!

Path: BEDABBCDC

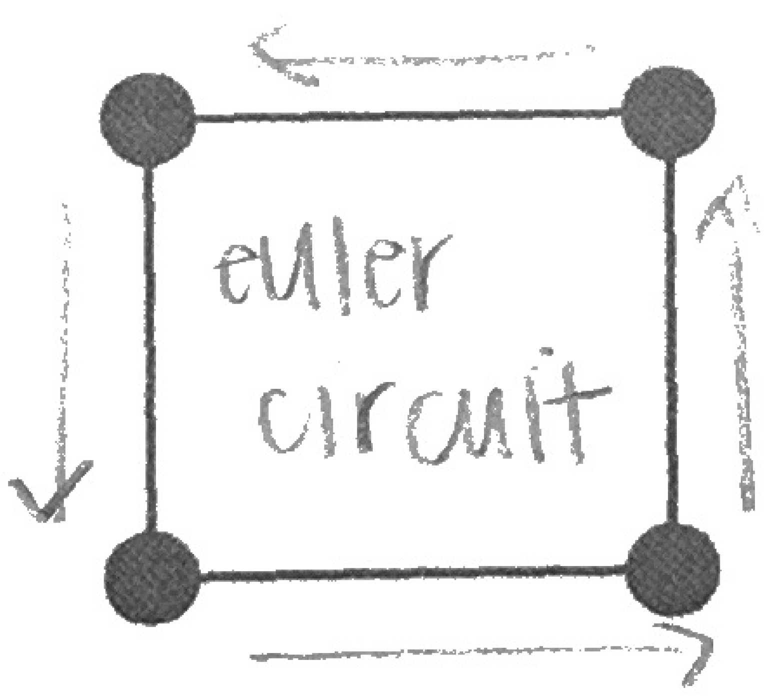


Euler Path: CDCBBEDAB

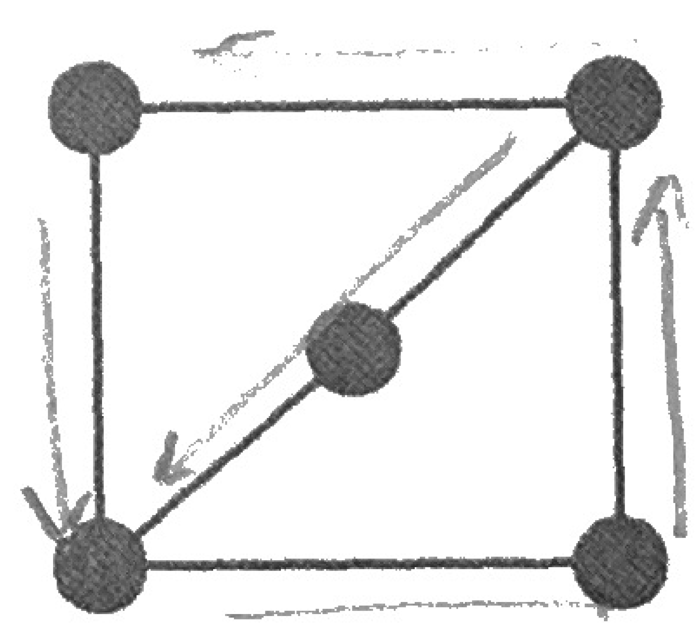
Euler Path: CDEBBADCB

* Euler Circuit: no circuit exists b/c 1 odd vertex

4. For the following graphs, decide which have Euler circuits and which do not.



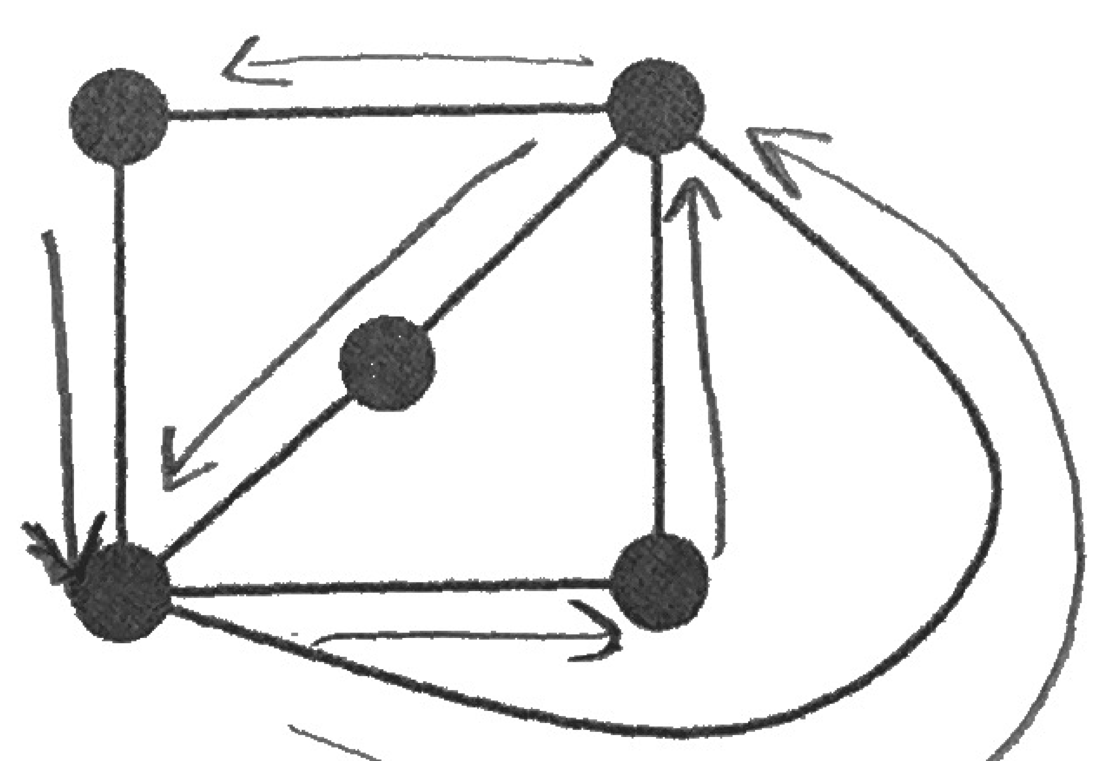
Graph I



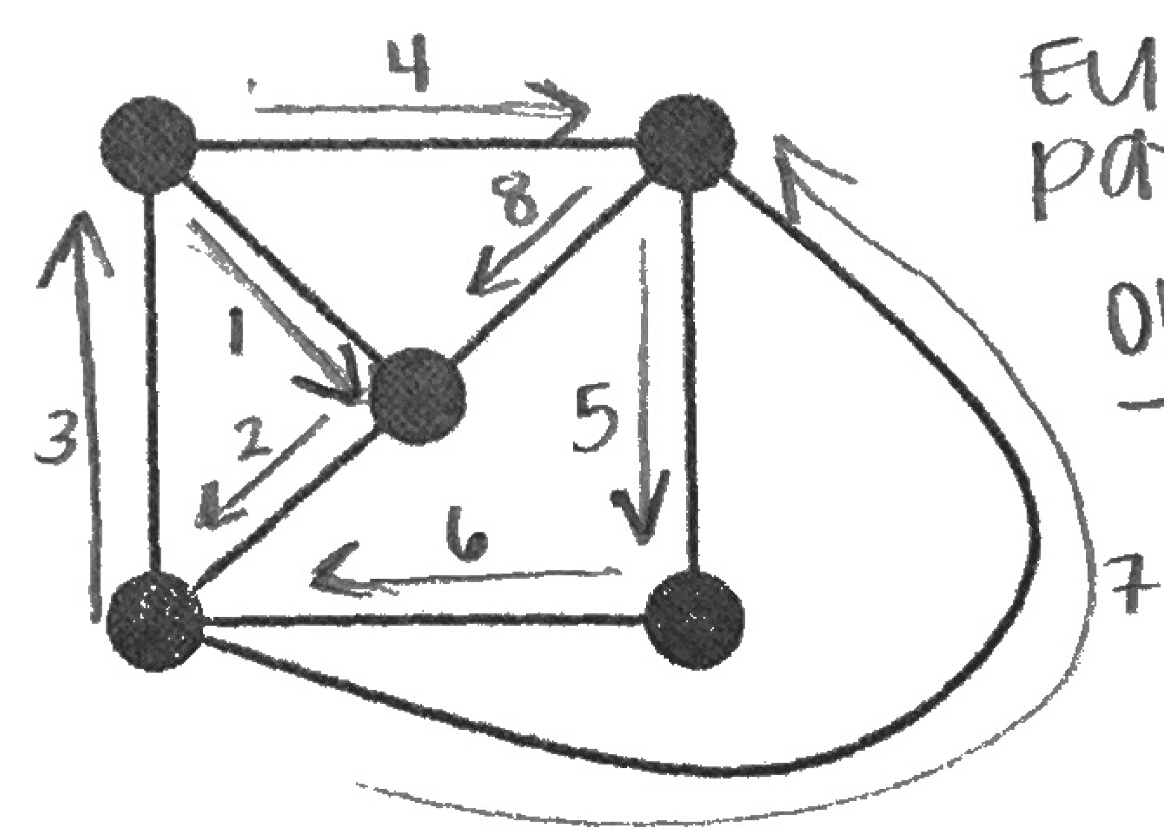
Graph II

Euler path only

Euler circuit



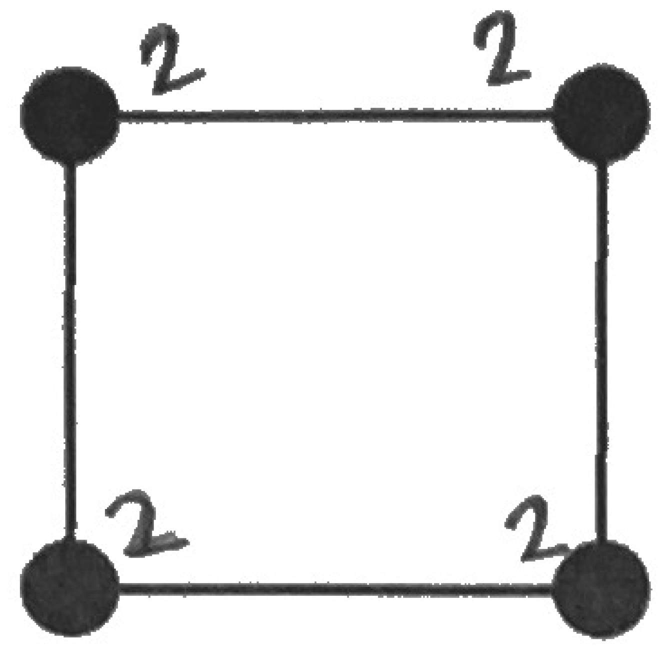
Graph III



Graph IV

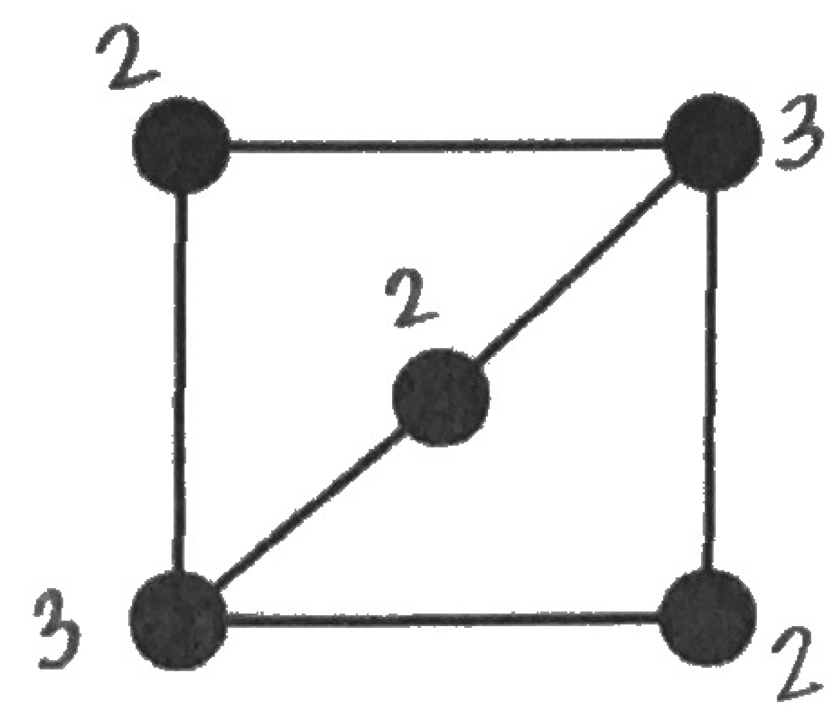
Euler path only

5. The degree of a vertex is the number of EDGES that meet at a VERTEX. Determine the degree of each vertex in Graphs I – IV.



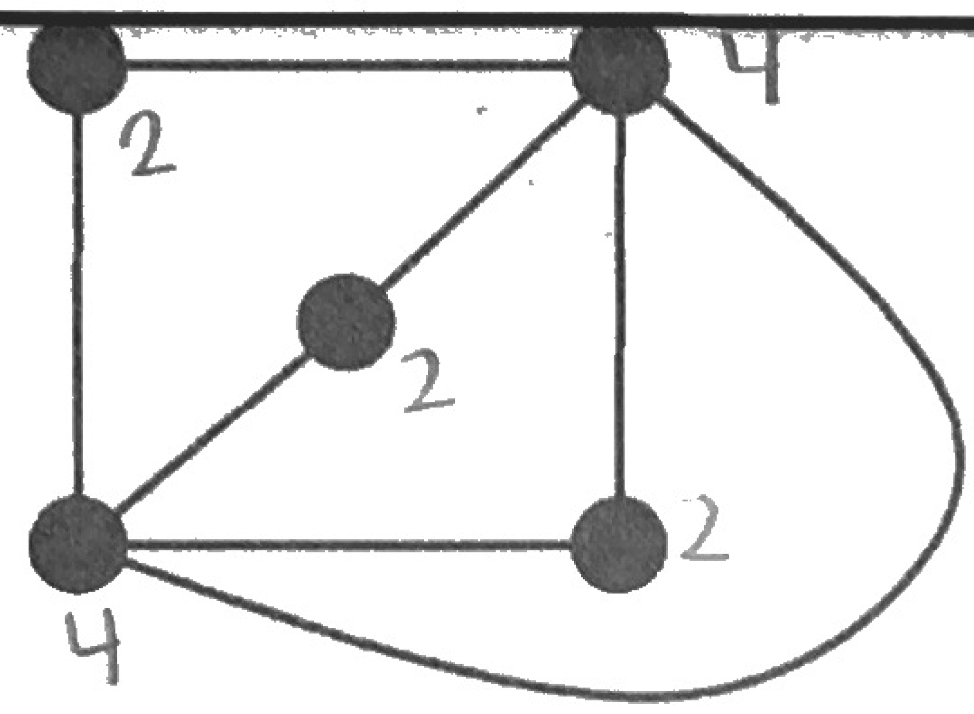
Graph I

Degree: 8



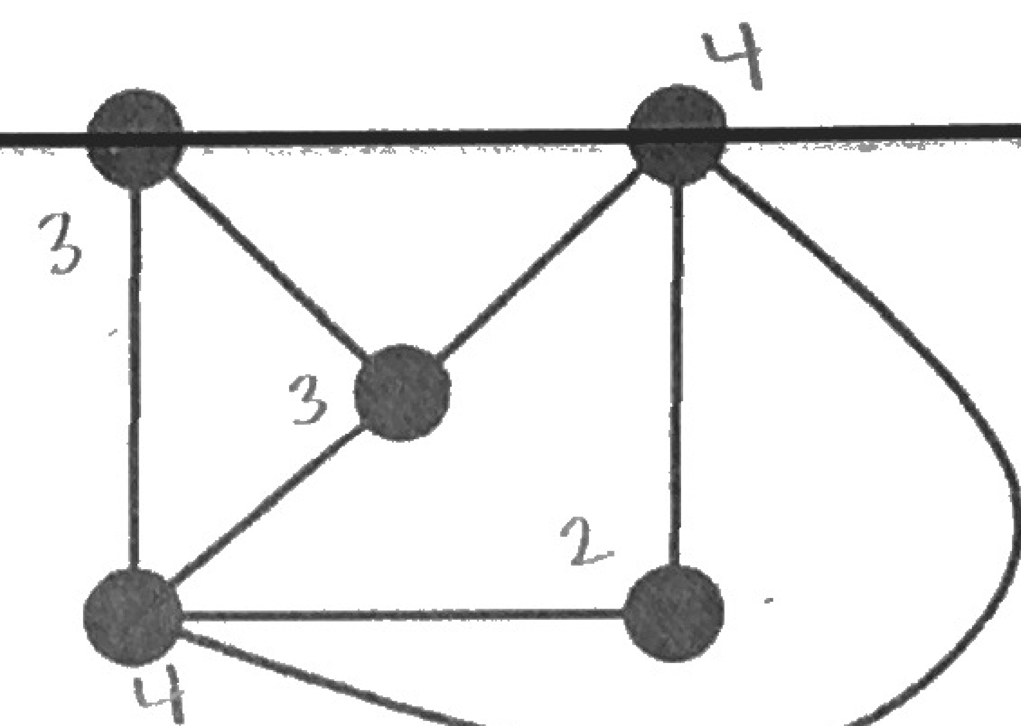
Graph II

Degree: 12



Graph III

Degree: 14



Graph IV

Degree: 16

6. For the graphs from Question 4 that are Euler Circuits, how many vertices have an odd degree?

None have odd vertices

7. For the graphs from Question 4 that are Euler Circuits, how many vertices have an even degree?

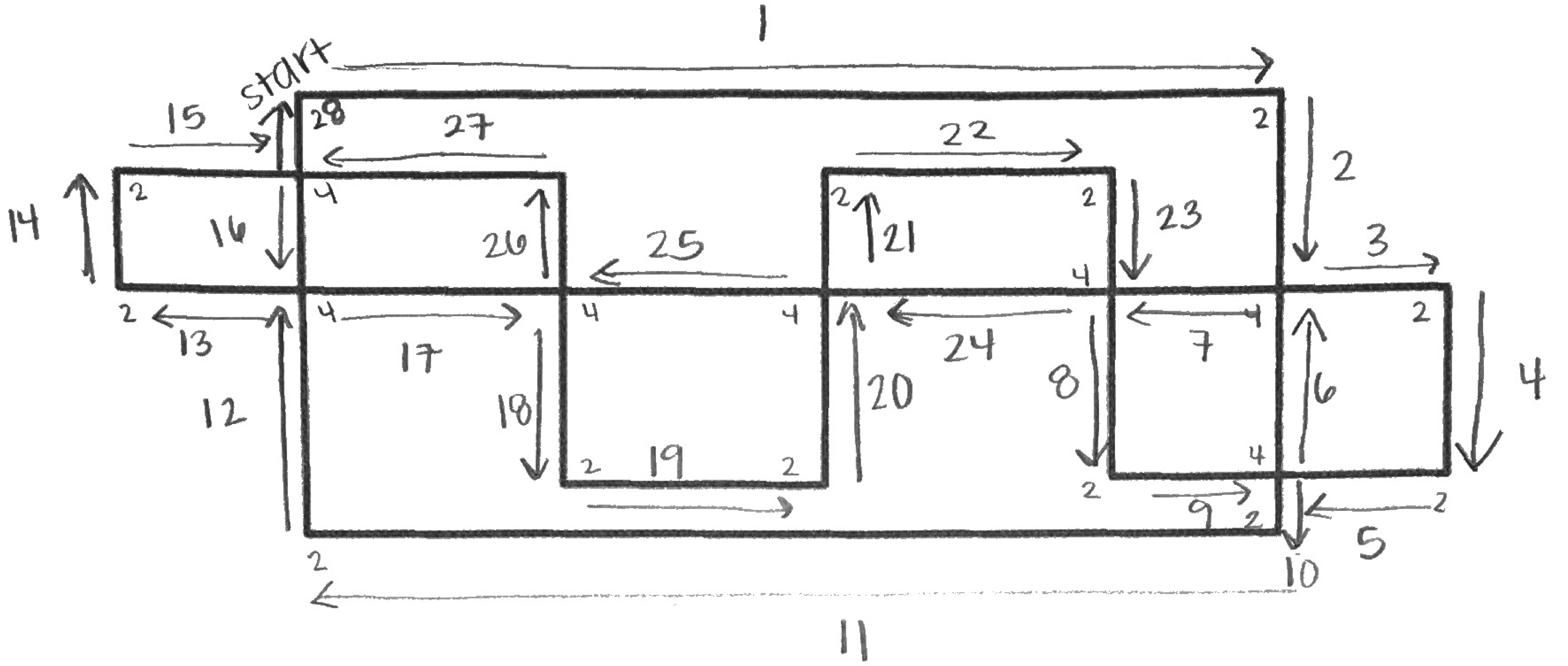
All of the vertices are even.

8. Form a **conjecture** (an opinion or idea formed without proof) about how you might quickly decide whether a graph is an Euler Circuit.

An Euler circuit has all even degree vertices. If you visit a vertex, you must be able to leave that vertex.

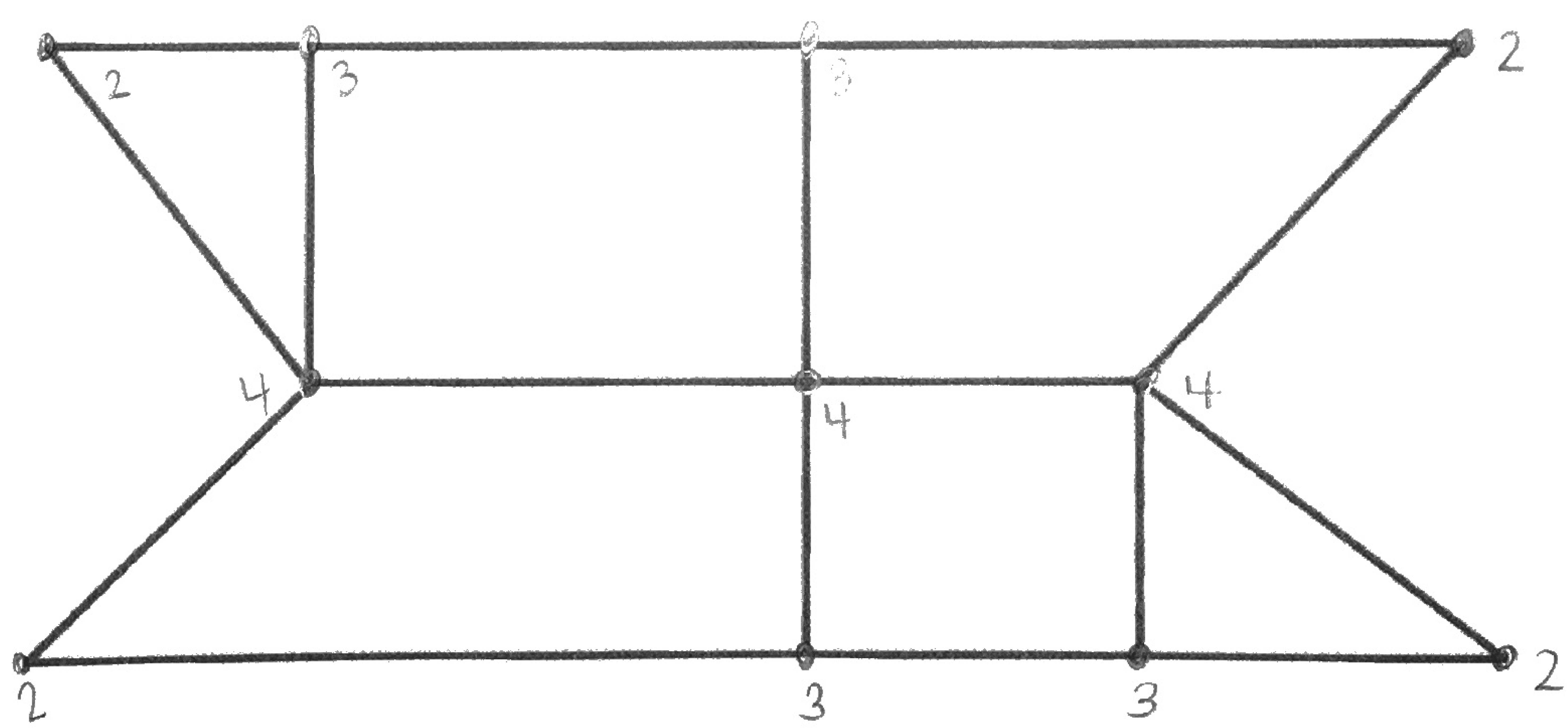
9. What does your conjecture tell you about the Konigsberg Bridge problem and the garden scenario n Question 2? The graph for Königsberg has vertices with an odd degree, therefore an Euler circuit doesn't exist. You can't stop & start at the same place. The western garden has 4 odd degrees so a circuit doesn't exist. The east has an Euler path but not a circuit.

10. Your friend Chet calls you on his cell phone and tells you that he has discovered a large rock embedded with gems! He is somewhere in your favorite hiking area, which has many interconnected paths, as shown below. Chet does not know exactly where he is, but he needs your help to carry the rock. To find him, you decide it would be most efficient to jog along all the paths in such a way that no path is covered twice. Find this efficient route on the map below or explain why no such route exists.



All even vertices — a circuit exists.

11. You have been hired to paint the yellow median stripe on the roads of a small town. Since you are being paid by the job and not by the hour, you want to find a path through the town that traverses each road only once. In the map of the town's roads below, find such a path or explain why no such path exists



An Euler circuit does not exist since several vertices have an odd degree. You cannot travel the entire town by crossing each street exactly once.