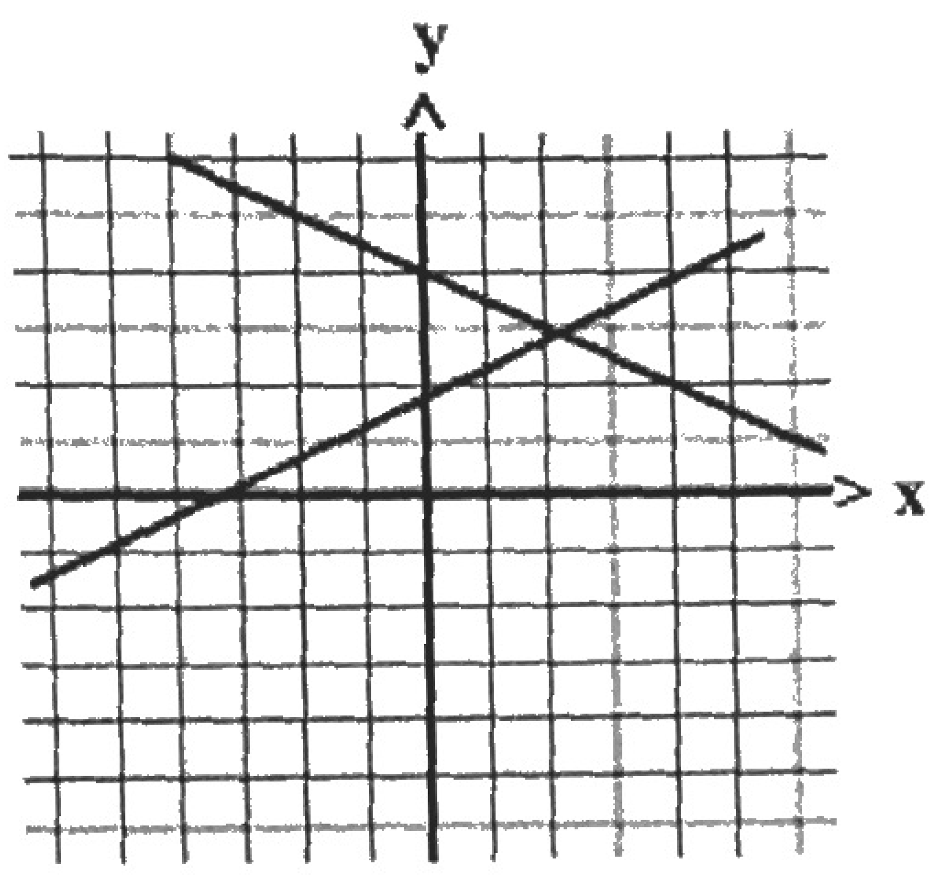
Name Key Date ____________ Period ____________

Notes: Solving Systems of Equations

A "*system of equations*" is a situation where two or more equations have two or more variables. Many times the variables are "x" and "y", but, of course, the variables could be anything. There are several methods for solving systems of equations. Here, we'll cover the following three:

1) Graphing 2) Substitution 3) Elimination

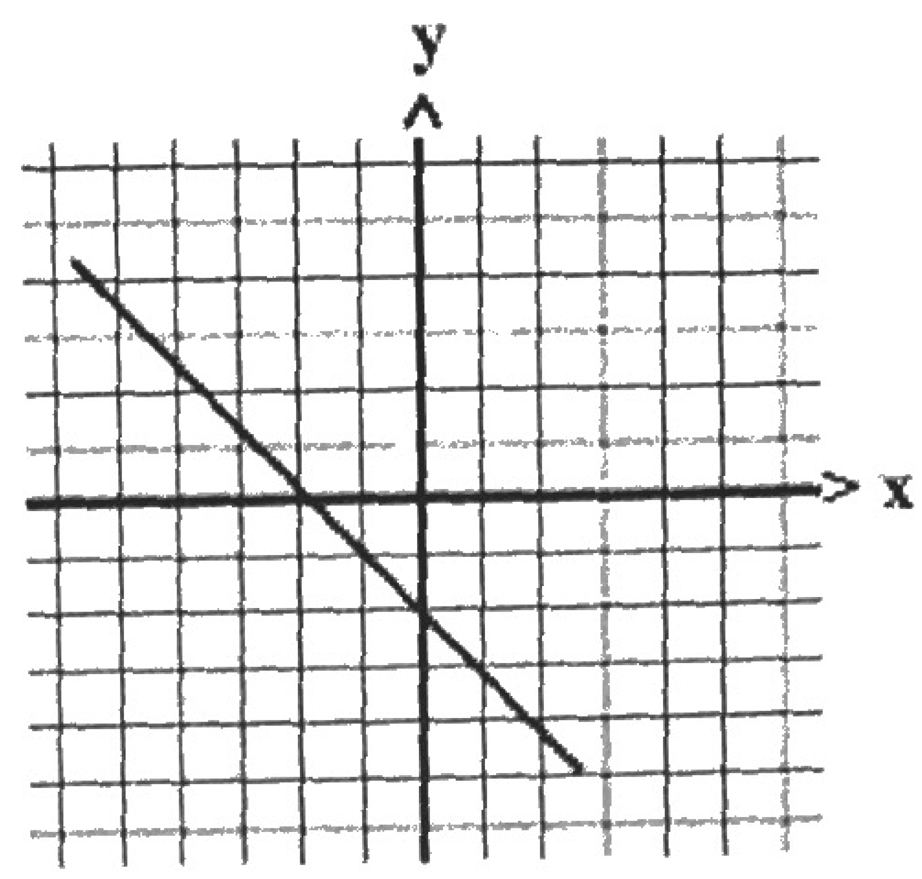
Method One: Graphing



graphs intersect

one point of intersection

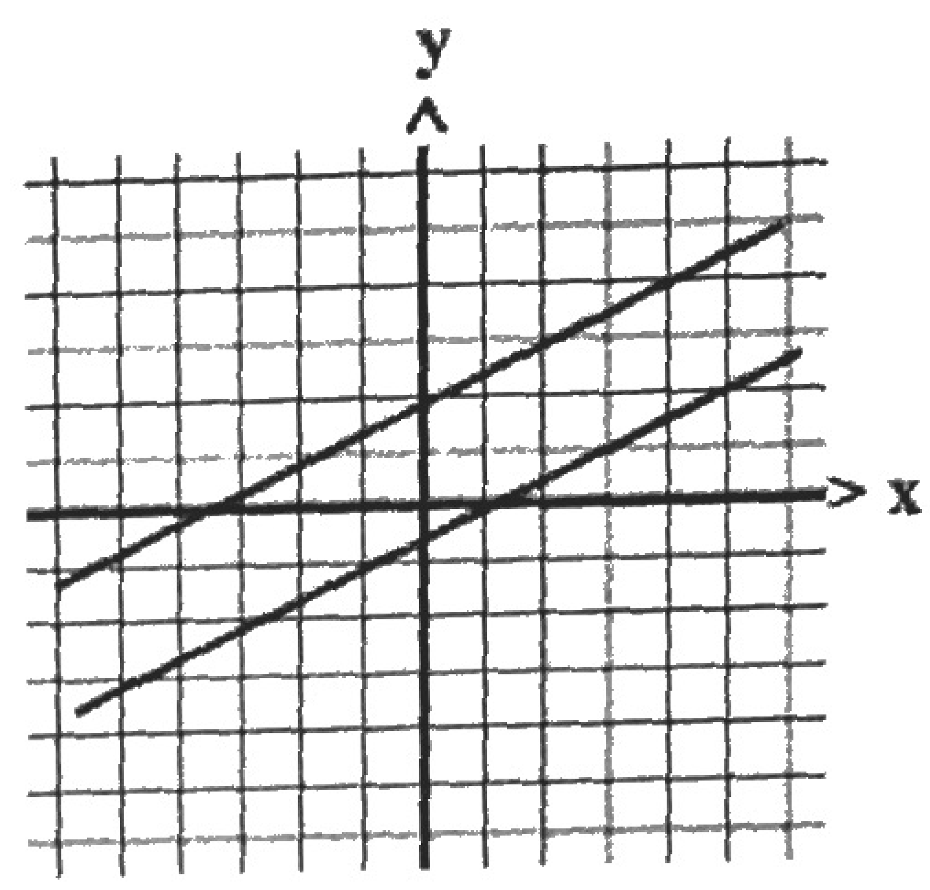
exactly one solution



graphs are identical

infinitely many points of intersection

infinitely many solutions



graphs are parallel

no points of intersection

no solutions

CLASSIFICATIONS

Method Two: Substitution

1. Identify a variable to isolate. (HINT: This step is easier if the variable does not have a coefficient.)
2. Rearrange one equation so that a variable is isolated.
3. Substitute and solve for the first variable.
4. Substitute and solve for the second variable.

Example 1: $2x + y = 16$ and $3x - 2y = -18$ — Original system of equations

$2x + y = 16$ — Get this y isolated because it doesn't have a coefficient. (Step 1)

$y = 16 - 2x$ — Move "$2x$" over so that y is isolated. (Step 2)

$3x - 2(16 - 2x) = -18$
$3x - 32 + 4x = -18$
$7x - 32 = -18$
$7x = 14$
$x = 2$

Substitute "$16 - 2x$" for "y" in the other equation. Then, just solve the equation for x. (Step 3)

$y = 16 - 2(2)$
$y = 16 - 4$
$y = 12$

Substitute "2" for "x" anywhere you'd like...just pick the place that looks the easiest...then solve for y. (Step 4)

(2, 12)

Sometimes, your answer will be written as an ordered pair.

Method Three: Elimination

1. "Line up" the two equations. Write one under the other and arrange them in the same order.
2. Look for the same coefficient (just the number...don't worry about the sign) of the same variable. If necessary, create this "same coefficient" situation by multiplication.
3. Either add or subtract (here's the rule: **S**ame **S**ign **S**ubtract) the entire equations so that one variable, the one with the same coefficient, is eliminated.
4. Solve for the first variable.
5. Substitute and solve for the second variable.

Example 1:	$5x + 9y = -35$ and $3x - 2y = 16$	Original system of equations
	$5x + 9y = -35$ $3x - 2y = 16$	"Line up" the equations. They're in the same order: *x* term, *y* term, equal sign, constant. (Step 1)
	$3(5x + 9y = -35)$ $5(3x - 2y = 16)$ ↓ $15x + 27y = -105$ $15x - 10y = 80$	Since the *x*'s have different coefficients, and the *y*'s have different coefficients, we need to create that situation by multiplication...the top equation by 3, and the bottom equation by 5. Then, the *x*'s will both have 15 as a coefficient. (Step 2)
	$37y = -185$	Since the *x*'s have the same coefficient, and they have the same sign, we'll subtract. (Step 3)
	$y = -5$	Solve for *y*. (Step 4)
	$3x - 2(-5) = 16$ $3x + 10 = 16$ $3x = 6$ $x = 2$	Substitute "–5" for *x* anywhere you'd like...just pick the place that looks easiest...and solve for *x*. (Step 5)
	(2, –5)	Answer is written as an ordered pair.

Example Problems:

Solve the following system of equations by **substitution:**

$5x + y = 9$ $\quad y = -5x + 9$
$10x - 7y = -18$

$10x - 7(-5x + 9) = -18$
$10x + 35x - 63 = -18$
$45x = 45$
$\boxed{x = 1}$

$y = -5(1) + 9$
$\boxed{y = 4}$

(1, 4)

Solve the following system of equations by **elimination:**

$8x + 14y = 4$ $\quad 8x + \cancel{14y} = 4$
$2(-6x - 7y = -10)$ $\quad -12x - \cancel{14y} = -20$

$-4x = -16$
$\boxed{x = 4}$

$8(4) + 14y = 4$
$32 + 14y = 4$
$14y = -28$
$\boxed{y = -2}$

(4, -2)

Name: Key
AMDM

Date: ____________

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Solving Systems Practice

Solving Systems of Equations Using the Addition/Elimination Method

THESE SYSTEMS OF EQUATIONS CAN BE SOLVED ALGEBRAICALLY BY USING THE ELIMINATION METHOD AS FOLLOWS:

A. Add the two equations to see if one variable cancels out.
B. If not, multiply one or both of the equations by a constant then add to eliminate one of the variables.
C. Solve for the remaining variable.
D. Substitute the value of this variable into one of the **original equations** and solve for the other variable.
E. Check your solution in both equations.
F. Write your answer as:
    1. An ordered pair,
    2. No solution, or
    3. With set notation (infinite solutions).

(elimination)

A. SOLVE THE FOLLOWING SYSTEMS OF EQUATIONS USING THE ADDITION METHOD, MULTIPLY ONE EQUATION IF NECESSARY.

1. $\cancel{x} + y = 1$
$\cancel{-x} + y = 5$
$2y = 6$
$y = 3$
$x + 3 = 1$
$x = -2$
$\boxed{(-2, 3)}$

2. $2x + \cancel{y} = -3$
$3x - \cancel{y} = -12$
$5x = -15$
$x = -3$
$2(-3) + y = -3$
$-6 + y = -3$
$y = 3$
$\boxed{(-3, 3)}$

3. $2x - y = 8$
$-2(x + 3y = 4)$
$\cancel{2x} - y = 8$
$-\cancel{2x} - 6y = -8$
$-7y = 0$
$y = 0$
$x + 3(0) = 4$
$x = 4$
$\boxed{(4, 0)}$

4. $-1(2x + 2y = 4)$
$-x + \cancel{2y} = 4$
$-2x - \cancel{2y} = -4$
$-3x = 0$
$\boxed{x = 0}$
$2(0) + 2y = 4$
$2y = 4$
$\boxed{y = 2}$
$\boxed{(0, 2)}$

5. $3x + 4y = 25$
$-(3x - 3y = -3)$
$\cancel{3x} + 4y = 25$
$-\cancel{3x} + 3y = 3$
$7y = 28$
$y = 4$
$3x + 4(4) = 25$
$3x + 16 = 25$
$3x = 9$
$x = 3$
$\boxed{(3, 4)}$

6. $-(x + y = 5)$
$\cancel{x} + \cancel{y} = -5$
$-\cancel{x} - \cancel{y} = -5$
$0 \neq -10$
$\boxed{\text{no solution}}$

YOU TRY:

A. $3x + \cancel{4y} = 10$
$6x - \cancel{4y} = 8$
$9x = 18$
$x = 2$
$3(2) + 4y = 10$
$4y = 4$
$y = 1$
$\boxed{(2, 1)}$

B. $x - 5y = 5$
$5(3x + y = 31)$
$x - \cancel{5y} = 5$
$15x + \cancel{5y} = 155$
$16x = 160$
$x = 10$
$3(10) + y = 31$
$y = 1$
$\boxed{(10, 1)}$

C. $2(x - 2y = -2)$
$-2x + 4y = 4$
$2x - 4y = -4$
$0 = 0$
infinitely many solutions

B. SOLVE BY MULTIPLYING BOTH EQUATIONS TO ELIMINATE ONE VARIABLE

$5(5x + 2y = 8)$ $25x + 10y = 40$
$2(3x - 5y = 11)$ $6x - 10y = 22$

$31x = 62$
$x = 2$

$5(2) + 2y = 8$
$2y = -2$
$y = -1$

$\boxed{(2, -1)}$

$3(3x + 2y = 8)$ $9x + 6y = 24$
$2(4x - 3y = -12)$ $8x - 6y = -24$

$17x = 0$
$x = 0$

$3(0) + 2y = 8$
$2y = 8$
$y = 4$

$\boxed{(0, 4)}$

$4(4x + 3y = 1)$ $16x + 12y = 4$
$3(5x - 4y = 9)$ $15x - 12y = 27$

$31x = 31$
$x = 1$

$4(1) + 3y = 1$
$3y = -3$
$y = -1$

$\boxed{(1, -1)}$

4. $-3(2x + 5y = 11)$ $-6x - 15y = -33$
$2(3x + 8y = 16)$ $6x + 16y = 32$

$y = -1$

$2x + 5(-1) = 11$
$2x = 16$
$x = 8$

$\boxed{(8, -1)}$

Of the three methods for solving systems of equations (graphing, substitution, addition/elimination) which method would work best in each of the following problems? Explain your choice!! Solve.

subst.

1. $y = 2x - 5$
$4y + 3x = 13$

$4(2x - 5) + 3x = 13$
$8x - 20 + 3x = 13$
$11x = 33$
$x = 3$

$y = 2(3) - 5$
$y = 1$

$\boxed{(3, 1)}$

elim.

2. $5(5x - 4y = 15)$ $25x - 20y = 75$
$4(2x + 5y = 6)$ $8x + 20y = 24$

$33x = 99$
$x = 3$

$2(3) + 5y = 6$
$5y = 0$
$y = 0$

$\boxed{(3, 0)}$

3. $y = \frac{2}{3}x - 5$
$y = -\frac{1}{3}x + 1$

subst.

$\frac{2}{3}x - 5 = -\frac{1}{3}x + 1$
$x = 6$

$y = -\frac{1}{3}(6) + 1$
$= -2 + 1 = -1$

$\boxed{(6, -1)}$

4. $2x = 3y + 6$
$2(5x - 2y = 15)$ elim
$-5(2x - 3y = 6)$

$10x - 4y = 30$
$-10x + 15y = -30$

$11y = 0$
$y = 0$

$2x = 3(0) + 6$
$2x = 6$
$x = 3$

$\boxed{(3, 0)}$